M.Sc. F.Y. (Physics) (NEP Pattern) Semester-I

NEP-235 / 01MSCPH3-DSC Paper-III : Mathematical Physics

P. Pages : 2 **GUG/W/23/15136**

Time : Three Hours * 8 0 6 3 * Max. Marks : 80

Either:

1. a) State and prove Cayley Hamilton theorem. **8**

b) Using Cayley Hamilton theorem, find A^{-1}, given that **8**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

OR

c) Find the eigenvalues and corresponding eigenvectors for the matrix. **10**

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

d) Define : **6**

i) Vector space and ii) Inner Product space

Either:

2. a) Find the Fourier series for f(x) if **8**

$$f(x) = -\pi, \quad -\pi < x < 0$$
$$= x, \quad 0 < x < \pi$$

b) Find Fourier sine transform of **8**

$$f(x) = \frac{e^{-ax}}{x}$$

OR

c) Find the Fourier series of the function **8**

$$f(x) = \begin{cases} -1 \text{ for } & -\pi < x < 0 \\ 1 \text{ for } & 0 < x < \pi \end{cases}$$

d) Find the Fourier transform of the following. **8**

$$f(x) = \begin{cases} 1 + \frac{x}{a}, & -a < x < 0 \\ 1 - \frac{x}{a}, & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$

Either:

3. a) Solve the differential equation by power series solution, $(1-x^2)y'' - 2xy' + 2y = 0$ about x = 0. **8**

b) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomial. **8**

OR

c) Prove that for Bessel's function $J_n(x)$, $J_{-n}(x) = (-1)^n \cdot J_n(x)$ **8**

d) Show that $\frac{1}{(1-t)} \exp\left(\frac{-t \cdot x}{1-t}\right)$ is the generating function of Laguerre polynomial. **8**

Either:

4. a) State and prove contraction theorem of tensor. **8**

b) Prove the following. **8**

i) $\text{grad}\ (\vec{f} \cdot \vec{g}) = \vec{f} \times \text{curl}\ \vec{g} + \vec{g} \times \text{curl}\ \vec{f} + \vec{f} \cdot \nabla \vec{g} + \vec{g} \cdot \nabla \vec{f}$ **8**

ii) $\text{grad}\ (\text{div} \cdot \bar{f}) = \text{curl}(\text{curl}\ \bar{f}) + \nabla^2 \bar{f}$

OR

c) Define Christoffel symbols of first and second kind and prove that **8**

$$\frac{\partial g^{p \cdot q}}{\partial x^{\bar{m}}} = -g^{p \cdot n} \left\{ \begin{matrix} q \\ m \cdot n \end{matrix} \right\} - g^{p.n} \left\{ \begin{matrix} p \\ m \cdot n \end{matrix} \right\}$$

d) What are metric Tensors? Obtain the components of metric tensor in three dimensional space in terms of spherical polar coordinates. **8**

5. Answer all the followings.

a) Prove that the matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. **4**

b) What are Dirichlet's condition for Fourier series. **4**

c) Prove that, $H'_{2n}(0) = 0$. **4**

d) Using Tensor analysis prove **4**

i) $\text{grad}\ (\phi\psi) = \phi\ \text{grad}\ \psi + \psi\ \text{grad}\ \phi$

ii) $\text{div}(\phi f) = \phi\, \text{div}\, f + f.\text{grad}\, \phi$

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