M.Sc. - I (Mathematics) (NEP Pattern) Semester-I

NEP-64-1 / DSE-1 - Numerical Analysis

P. Pages : 2 **GUG/W/23/15115**

Time : Three Hours * 8 0 4 2 * Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. Each Questions carries equal marks.

UNIT – I

1. a) Let $g(x)$ be continuous on $[a,b]$, and assume $g([a,b]) \subset [a,b]$ further more, assume there is a constant $0 < \lambda < 1$, with $|g(x) - g(y)| \le \lambda |x - y| \forall x, y \in [a,b]$
Then prove that $x = g(x)$ has a unique solution α in [a, b]. Also the iterates $x_n = g(x_{n-1}),\ n \ge 1$ will converge, to α for any choice of x_0 in $[a,b]$ and
$$|\alpha - x_n| \le \frac{\lambda^n}{1-\lambda}|x_1 - x_0|.$$ **8**

b) Find the root of the equation $y(x) = x^3 - 2x - 5 = 0$
Which lies between 2 and 3 by using Muller's method. **8**

OR

c) Apply Newton's method to the following function. **8**
$$f(x) = \begin{cases} x^{2/3}, & x \ge 0 \\ -x^{2/3}, & x < 0 \end{cases}$$
With the root $\alpha = 0$. What is the behaviour of the iterates? Do they converge and if so, at what rate?

d) Discuss the Muller's method? **8**

UNIT – II

2. a) Prove that for $k \ge 0, f[x_0, x_1, ..., x_k] = \frac{1}{k!h^k}\Delta^k f_0$ **8**

b) Find the Polynomial of degree ≤ 2 that passes through the points $(0,1), (-1,2)$ and $(1,2)$? **8**

OR

c) State and prove Hermite-Genocchi theorem? **8**

d) Show that for any two functions f and g for any two constant α and β **8**
$$\Delta^r(\alpha f(x) + \beta g(x)) = \alpha \Delta^r f(x) + \beta\, \Delta^r g(x),\ r \ge 0$$

UNIT – III

3. a) Let $f(x)$ be continuous for $a \le x \le b$ and let $\in > 0$. Then prove that there is a polynomial $p(x)$ for which $|f(x) - p(x)| \le \in, a \le x \le b$. **8**

b) Discuss the Gram-Schmidt theorem. **8**

OR

c) Prove that for $f; g \in c[a, b]$ **8**

i) $|(f, g)| \le \|f\|_2 \|g\|_2$

ii) $\|f + g\|_2 \le \|f\|_2 + \|g\|_2$

d) Find linear least square approximation of the function. **8**

$f(x) = e^x \text{on} -1 \le x \le 1$

UNIT – IV

4. a) Evaluate $I(f) = \int_0^1 \frac{dx}{1+x}$ by using simple Simpson's rule? **8**

b) Derive Newton-cotes integration formula for n = 1. **8**

OR

c) Obtain the expression for Peano-Kernel error formula. **8**

d) Obtain the composite trapezoidal rule with error. Find the expression for the asymptotic error? **8**

5. a) Consider Newton's method for finding the positive square root of a > 0. **4**

Derive $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$

b) Prove that $\Delta^r f(x_i) = h^r f^{(r)}(\xi_i)$ for some $x_i < \xi_i \le x_i + r$ **4**

c) Discuss the Minimax Approximation problem. **4**

d) Evaluate $\int_0^1 \frac{e^x - 1}{x} dx$. **4**

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