M.Sc. - I (Mathematics) (NEP Pattern) Semester-I

NEP-62 / DSC-2 - Paper-II : Topology

P. Pages : 2 **GUG/W/23/15113**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that the union of a denumerable number of denumerable sets is a denumerable set. **8**

b) Prove that $2^a > a$ for every cardinal number a. **8**

OR

c) Prove that the set of all real numbers is uncountable. **8**

d) Prove that every infinite set contains a denumerable subset. **8**

UNIT – II

2. a) Let X = 1N, the set of positive integers, and let τ be the family consisting of ϕ, X, and all subsets of the form {1, 2,…..,n). Show that τ is a topology for X. **8**

b) For any set E in a topological space, prove that $c(E) = E \bigcup d(E)$. **8**

OR

c) Let (X, τ) be a topological space & X^* is a subset of X. Then prove that τ^* is a topology for X^*, where τ^* is an inducted or relative topology for X^*. **8**

d) Prove that a family β of sets is a base for a topology for the set $X = \bigcup\{B : B \in \beta)$. If and only if for every $B_1, B_2 \in \beta$ and every $x \in B_1 \bigcap B_2$, there exists a $B \in \beta$ such that $x \in B \subseteq B_1 \bigcap B_2$. **8**

UNIT – III

3. a) If C is a connected subset of a topological space (X, τ) which has a separation $X = A | B$, then prove that either $C \subseteq A$ or $C \subseteq B$. **8**

b) If f is a continuous mapping of (X, τ) into (X^*, τ^*), then prove that f maps every compact subset of X onto a compact subset of X*. **8**

OR

c) If f is a homeomorphism of X onto X*, then prove that f maps every isolated subset of X onto an isolated subset of X*. **8**

d) Prove that a compact subset of a topological space is countably compact. **8**

UNIT – IV

4. a) Prove that a topological space X is a To-space iff the closures of distinct points are distinct. **8**

b) Prove that a topological space X satisfying the first axiom of countability is a Hausdorff space iff every convergent sequence has a unique limit. **8**

OR

c) Prove that a topological space X is regular iff for every point $x \in X$ and open set G containing x there exists an open set G^* such that $x \in G^*$ and $c(G)^* \subseteq G$. **8**

d) Prove that in a $T_1 -$space X, a point x is a limit point of a set E iff every open set containing x contains an infinite number of distinct points of E. **8**

5. a) Define : cardinal number, & sum & product of cardinal n umbers. **4**

b) Define : Topological space and open sets. **4**

c) Define : Continuous functions and homeomorphisms. **4**

d) Define : First axiom space & second axiom space. **4**

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