PSCMTH19C - Combinatorics

P. Pages : 2

Time : Three Hours

2508

GUG/S/23/13772

Max. Marks : 100

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Note:
i) Solve all the **five** questions.
ii) Each questions carries equal marks.

UNIT – I

1. a) A committee of K people is to be chosen from a set of seven women and four men. How many ways are there to form the committee if **10**
   a) The committee consists of three women and two men?

   b) The committee has four people and one of them must be Mr. Baggins?

   c) The committee has four people and at least two are women?

   d) The committee has four people, two of each sex, and Mr. and Mrs. Baggins cannot both be on the committee?

   b) How many arrangements of the seven letters in the word SYSTEMS have the E occurring somewhere before the M? How many arrangements have the E somewhere before the M and the three S5 grouped consecutively? **10**

OR

   c) There are five different Spanish books, six different French books, and eight different Transylvanian books. How many ways are there to pick an unordered pair of two books not both in the same language? **10**

   d) What is the probability that a 4-digit campus telephone number has one or more repeated digits? **10**

UNIT – II

2. a) Find the coefficient of x^{16} in $\left(x + x^2 + x^3 + x^4 + x^5\right)\left(x^2 + x^3 + x^4 + - - -\right)^5$. **10**

   b) How many ways are there to select 25 toys from seven types of toys with between two and six of each type? **10**

OR

   c) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? **10**

   d) Find the coefficient of x^{12} in $\left(1 - 4x\right)^{-5}$. **10**

UNIT – III

3. a) Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = a_1 = 1$. **10**

b) Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}$ with $a_0 = a_1 = 1$. **10**

OR

c) Solve the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}$ with $a_0 = a_1 = 1$. **10**

d) Solve the recurrence relation $a_n = 3a_{n-1} - 4n + 3x2^n$ to find its general solution. Also find the solution when $a_1 = 8$. **10**

UNIT – IV

4. a) State & prove Inclusion – Exclusion formula. **10**

b) How many ways are there to select a 6 – card hand from a regular 52 – card-deck such that the hand contains at least one card in each suit? How many 6-card hands with a void in at least one suit? **10**

OR

c) How many positive integers ≤ 90 are relatively prime to 90? **10**

d) How many ways are there to distribute r distinct objects into five distinct boxes with at least one empty box? **10**

5. a) How many different ways are there to select six hot dogs from three varieties of hot dog? **5**

b) Find the generating function for a_r, the number of ways to select r balls from three green, three white, three blue, and three gold balls. **5**

c) Every year Dr. Finch's rabbit population doubles. He started with six rabbits. How many rabbits does he have after eight years? After n years? **5**

d) If a school has 100 students with 40 taking French, 40 taking Latin, and 40 taking German, 20 students are taking any given pair of languages, and 10 students are taking all three languages, then how many students are taking no language? **5**

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