M.Sc.(Mathematics) New CBCS Pattern Semester-IV

PSCMTH19C - Combinatorics

P. Pages : 2 | **GUG/W/23/13772**

Time : Three Hours | * 7 3 5 8 * | Max. Marks : 100

Notes : 1. Solve all the five questions.
2. Each question carries equal marks.

UNIT – I

1. a) How many ways are there to form a three-letter sequence using the letters a, b, c, d, e, f. **10**

i) With repetition of letters allowed?

ii) Without repetition of any letter?

iii) Without repetition of and containing the letter e?

iv) With repetition and containing e?

b) How many ways are there to arrange the seven letters in the word SYSTEMS? In how many of these arrangements do the three Ss appear consecutively? **10**

OR

c) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 12$, with $x_i \geq 0$? How many solutions with $x_i \geq 1$? How many solutions with, $x_1 \geq 2,\ x_2 \geq 2,\ x_3 \geq 4,\ x_4 \geq 0$? **10**

d) What is the probability that a 4-digit campus telephone number has one or more repeated digits? **10**

UNIT – II

2. a) Use generating functions to find the number of ways to collect \$15 from 20 distinct people if each of the first 19 people can give a dollar or nothing and the twentieth person can given either \$1 or \$5 or nothing. **10**

b) Find the number of ways to place 25 people into two rooms with at least one person in each room. **10**

OR

c) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? **10**

d) Find the coefficient of x^{47} in **10**

$$\left(x^{10} + x^{11} + ---- + x^{25}\right)\left(x + x^2 + ---- + x^{15}\right)\left(x^{20} + --- + x^{45}\right)$$

UNIT – III

3. a) Solve the recurrence relation **10**
$a_n = a_{n-1} + n(n-1), a_0 = 3$

b) Solve the recurrence relation **10**
$a_n = a_{n-2}$ with $a_0 = a_1 = 1$

OR

c) Solve the recurrence relation **10**
$a_n = 2a_{n-1} - a_{n-2}$, with $a_0 = a_1 = 2$

d) Solve the recurrence relation **10**
$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, with $a_0 = a_1 = 1$, $a_2 = 2$

UNIT – IV

4. a) How many positive integers ≤ 70 are relatively prime to 70? **10**

b) Let $A_1, A_2, ---, A_n$, be n sets in a universe u of N elements. Let S_K denote the sum of the sizes of all k-tuple intersections of the A_i s. **10**
Then prove that
$N(\bar{A}_1 \cap \bar{A}_2 \cap --- \cap \bar{A}_n) = N - S_1 + S_2 - S_3 + --- + (-1)^K S_K + --- + (-1)^n S_n$.

OR

c) How many ways are there to distribute r distinct objects into five distinct boxes with at least one empty box? **10**

d) How many different integer solutions are there to the equations **10**
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20$,
$0 \leq x_i \leq 8$

5. a) How many arrangements of MATHEMATICS are there? **5**

b) Find the coefficient of x^{32} in $\left(x^3 + x^4 + x^5 + x^6 + x^7\right)^7$ **5**

c) Find a recurrence relation for the number of ways to arrange n distinct objects in a row. Find the number of arrangements of eight objects. **5**

d) If a school has 100 students with 50 students taking French, 40 students taking Latin, and 20 students taking both languages, how many students take no language? **5**

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