M.Sc.-II (Mathematics) (New CBCS Pattern) Semester - IV

PSCMTH18 - Integral Equations

P. Pages : 2 **GUG/S/23/13769**

Time : Three Hours * 2 5 0 5 * Max. Marks : 100

Notes : 1. Solve all the questions.
2. Each question carry equal marks.

UNIT – I

1. a) Show that the function $u(x) = xe^{x}$ is a solution of the Volterra integral equation. **10**

$$u(x) = \sin x + 2\int_0^x \cos(x-t)\,u(t)\,dt$$

b) Form integral equation for $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with initial conditions $y(0) = 1, y'(0) = 1$. **10**

OR

c) Convert $\frac{d_2y}{dx^2} + xy = 1, y(0) = 0, y(1) = 1$ into an integral equation. **10**

d) Obtain integral equation from y”+y=0 with initial conditions $y(0) = y'(0) = 0$ **10**

UNIT – II

2. a) Find the eigen values & eigen functions of the homogeneous equation **10**

$$u(x) = \lambda\int_1^2 \left(xt + \frac{1}{xt}\right)u(t)\,dt$$

b) Solve the equation **10**

$$u(x) = \lambda\int_{-1}^{1} \left(5xt^3 + 4x^2t + 3tx\right)u(t)\,dt.$$

OR

c) Solve the equation $u(x) = f(x) + \lambda\int_{-1}^{1} \left(xt + x^2t^2\right)u(t)\,dt$ **10**

d) Solve the integral equation **10**

$$u(x) = \frac{6}{5}(1-4x) + \lambda\int_0^1 (x\log t - t\log x)\,u(t)\,dt.$$

UNIT – III

3. a) State & prove the Hilbert's theorem. **10**

b) By Hilbert – Schmidt theorem solve the equation **10**

$$u(x) = (x+1)^2 + \int_{-1}^{1} \left(xt + x^2t^2\right) u(t)\,dt$$

OR

c) Solve $u(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1}\left(xt + x^2t^2\right)u(t)\,dt$. **10**

d) Solve $u(x) = \cos 3x + \lambda \int_0^{\pi} \cos(x+t)\,u(t)\,dt$. **10**

UNIT – IV

4. a) Solve : $u(x) = f(x) + \frac{1}{2}\int_0^1 e^{x-t} u(t)\,dt$ **10**

b) Solve $u(x) = 1 + \lambda \int_0^{\pi} \sin(x+t)\,u(t)\,dt$ **10**

OR

c) Solve $u(x) = 1 + \int_0^{x} u(t)\,dt,\ u_0(x) = 0$ by successive approximation method. **10**

d) Solve $u(x) = f(x) + \frac{1}{2}\int_0^1 e^{x-t} u(t)\,dt$. **10**

5. a) Show that u(x) = 1 is a solution of the Fredholm integral equation. **5**

$$u(x) = e^x - x - \int_0^1 x\left(e^{xt} - 1\right)u(t)\,dt.$$

b) Define orthogonal functions & separable Kernel. **5**

c) Show that eigen values of symmetrical kernel are real. **5**

d) Find $k_2(x,t)$ for kernel **5**

$$k(x,t) = (1+x)(1-t),\ a = -1,\ b = 0.$$

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