M.Sc.- II (Mathematics) New CBCS Pattern Semester-IV

# PSCMTH18 : Integral Equations

P. Pages : 2 | GUG/W/23/13769

Time : Three Hours | Max. Marks : 100

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Notes : 1. Solve all the questions.
2. Each question carry equal marks.

## UNIT – I

**1.** a) Show that $u(x)=\frac{1}{2}$ is the solution of the integral equation $\int_0^x \frac{u(t)}{\sqrt{x-t}}\,dt=\sqrt{x}$. **10**

b) Obtain integral equation to the differential equation $y'''-2xy=0$ with conditions $y(0)=\frac{1}{2}, y'(0)=1, y''(0)=1$. **10**

**OR**

c) Convert the equation $y''+y=\cos x$ to Volterra integral equation with initial conditions $y(0)=0, y'(0)=1$ **10**

d) Show that $u(x)=\left(1+x^2\right)^{-3/2}$ is solution of the Volterra equation **10**

$$u(x)=\frac{1}{1+x^2}-\int_0^x \frac{t}{\left(1+x^2\right)}u(t)\,dt$$

## UNIT – II

**2.** a) Solve $u(x)=x+\lambda\int_0^1\left(xt^2+tx^2\right)u(t)\,dt$ **10**

b) Solve $u(x)=x+\lambda\int_0^{\pi}(1+\sin x\sin t)u(t)\,dt$ **10**

**OR**

c) Solve the integral equation. **10**

$$u(x)=f(x)+\lambda\int_0^1 (x+t)u(t)\,dt$$

d) Solve $u(x)=\cos x+\lambda\int_0^{\pi}\sin x\, u(t)\,dt$ **10**

## UNIT – III

**3.** a) Solve the homogeneous Fredholm equation. **10**

$f(x) = \lambda \int_0^1 e^x e^t f(t)\,dt$ using Schmidt solution.

b) Prove that the set of eigen values of the second iterated Kernel coincide with the set of the eigen values of the given Kernel. **10**

**OR**

c) Solve $u(x) = e^x + \lambda \int_0^1 (5x^2 - 3) t^2 u(t)\,dt$ **10**

d) Solve $u(x) = 2x + \lambda \int_0^1 \sin(\log x) u(t)\,dt$ **10**

## UNIT – IV

**4.** a) Find the resolvent Kernel of the Volterra integral equation with Kernel. **10**

$K(x,t) = \dfrac{2+\cos x}{2+\cos t}$

b) Solve $u(x) = \dfrac{5x}{6} + \dfrac{1}{2}\int_0^1 x\,t\,u(t)\,dt$ **10**

**OR**

c) Solve $u(x) = \sin x + 2\int_0^x e^{x-t} u(t)\,dt$ **10**

d) Solve $u(x) = x + \int_0^x (t-x) u(t)\,dt$ **10**

**5.** a) Define the eigen values & eigen functions for the integral equation. **5**

b) Define index of the eigen value & discuss the case $F(x) = 0$ for the Fredholm equation with separable Kernel. **5**

c) State the Riesz-Fischer theorem. **5**

d) Find $k_3(x,t)$ for the Kernel $k(x,t) = (1+x)(1-t), a = -1, b = 0$. **5**

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