M.Sc.-II (Mathematics) (New CBCS Pattern) Semester - IV

PSCMTH17 - Partial Differential Equations

P. Pages : 3 **GUG/S/23/13768**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that Singular integral is also a solution of the first order partial differential equation. **10**

b) Find the general solution of **10**

$$x(y^2-z^2)p-y(z^2+x^2)q=(x^2+y^2)z$$

OR

c) Prove that a necessary and sufficient condition that the Pfaffian differential equation **10**

$\overline{X}\cdot d\overline{r}=P(x,y,z)\,dx+Q(x,y,z)\,dy+R(x,y,z)\,dz=0$ be integrable is that $(\overline{X}\cdot \text{curl}\,\overline{X})=0$.

d) Find a complete integral of the partial differential equation. **10**

$$f=x^2p^2+y^2q^2-4=0$$

UNIT – II

2. a) Solve the initial value problem for the quasi-linear equation $z_x+z_y=1$ containing the initial data curve $c: x_0=s, y_0=s, z_0=\frac{1}{2}s$ for $0\le s\le 1$. **10**

b) Find the integral surface of the equation **10**

$$(2xy-1)p+(z-2x^2)q=2(x-yz)$$

Which passes through the line $x_0(s)=1, y_0(s)=0$ and $z_0(s)=s$.

OR

c) Find by the method of characteristics, the integral surface of $pq=xy$ which passes through the curve $z=x,\ y=0$. **10**

d) Consider the partial differential equation $f(x,y,z,p,q)=0$ where f has continuous second order derivatives with respect to its variables x, y, z, p and q, and at every point either $f_p\ne 0$ or $f_q\ne 0$. Suppose that the initial values $z=z_0(s)$ are specified along the initial curve $\Gamma_0: x=x_0(s), y=y_0(s), a\le s\le b$, where $x_0(s), y_0(s)$ and $z_0(s)$ have continuous second order derivatives. Suppose $p_0(s)$ and $q_0(s)$ have been determined such that **10**

$f(x_0(s), y_0(s), z_0(s), p_0(s), q_0(s)) = 0$ and $\frac{dz_0}{ds} = p_0 = \frac{dx_0}{ds} + q_0 \frac{dy_0}{ds}$, where p_0 and q_0 are continuously differentiable functions of s. If, in addition, the five functions x_0, y_0, z_0, p_0 and q_0 satisfy $f_q \frac{dx_0}{ds} - f_p \frac{dy_0}{ds} \neq 0$ then prove that in some neighbourhood of each point of the initial curve there exists one and only one solution $z = z(x, y)$ of $f(x, y, z, p, q) = 0$ such that $z(x_0(s), y_0(s)) = z_0(s), z_x(x_0(s), y_0(s)) = p_0(s), zy(x_0(s), y_0(s)) = q_0(s)$.

UNIT – III

3. a) Derive an equation governing small transverse vibrations of an elastic string. **10**

b) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to a Canonical form. **10**

OR

c) Obtain D'Alembert's solution of the one dimensional wave equation which describes the vibrations of an infinite string. **10**

d) Prove that for the equation **10**

$$Lu = u_{xy} + \frac{u}{4} = 0$$

The Riemann function is
$v(x, y; \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$, where J_0 is the Bessel's function of the first kind of order zero.

UNIT – IV

4. a) Suppose that u(x, y) is harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$. Then prove that u attains its maximum on the boundary B of D. **10**

b) Find the solution of the problem **10**

$$\nabla^2 u = 0, \ -\infty < x < \infty, \ y > 0$$
$$u(x, 0) = f(x), \ -\infty < x < \infty,$$

Such that u is bounded as $y \to \infty, u$ and u_x vanish as $|x| \to \infty$.

OR

c) Solve $u_t = u_{xx}, \ 0 < x < \ell, t > 0$ **10**

$$u(0, t) = u(\ell, t) = 0,$$
$$u(x, 0) = x(\ell - x), \ 0 \leq x \leq \ell$$

d) Show that the surfaces **10**

$$x^2 + y^2 + z^2 = cx^{2/3}$$

Can form an equipotential family of surfaces, and find the general form of the potential function.

5. a) Find a complete integral of **5**

$$f(p,q) = p + q - pq = 0$$

b) Discuss the method to find integral surface a semi – linear partial differential equation. **5**

c) Define: **5**

i) Second – order semi – linear partial differential equation

ii) Regular solution of second order semi – linear p.d.e.

d) State: **5**

i) The Dirichlet problem

ii) The Neumann problem

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