M.Sc.- II (Mathematics) New CBCS Pattern Semester-IV

PSCMTH17 : Partial Differential Equations

P. Pages : 2 **GUG/W/23/13768**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let z=f(x, y, a) be a one-parameter family of solutions of the first order partial differential equation $f(x,y,z,p,q)=0$. Then show that the envelope of this one parameter family, if it exists, is also a solution of the partial differential equation. **10**

b) Find the general integral of the partial differential equation. **10**
$$2x\left(y+z^2\right)p+y\left(2y+z^2\right)q=z^3$$

OR

c) Let $u(x,y)$ and $\nu(x,y)$ be two functions of x and y such that $\frac{\partial \nu}{\partial y}\neq 0$. If further , $\frac{\partial(u,\nu)}{\partial(x,y)}=0$, then prove that there exists a relation $F(u,\nu)=0$ between u and ν not involving x and y explicitly. **10**

d) Find a complete integral of the equation $p^2x+q^2y=z$ by Jacobi's method. **10**

UNIT – II

2. a) Solve the Cauchy problem for $2z_x+yz_y=z$ for the initial data curve **10**
$$c: x_0=s,\ y_0=s^2,\ z_0=s,\ 1\le s\le 2$$

b) Prove that if an element (x_0,y_0,z_0,p_0,q_0) is common to both an integral surface $z=z(x,y)$ and a characteristics strip, then the corresponding characteristics curve lies completely on the surface. **10**

OR

c) Find a complete integral of the equation **10**
$$\left(p^2+q^2\right)x=pz,$$
and the integral surface containing the curve.
$$c: x_0=0,\ y_0=s^2,\ z_0=2s$$

d) Find the solution of $z=p^2-q^2$ which passes through the curve **10**
$$c: x_0=s,\ y_0=0,\ z_0=-\frac{1}{4}s^2$$

UNIT – III

3. a) Derive the second order partial differential equation which describes the temperature distribution in a homogeneous isotropic solid. **10**

b) Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form. **10**

OR

c) Obtain D'Alembert’s solution of the one dimensional wave equation which describes the vibrations of a semi-infinite string. **10**

d) Solve **10**

$$y_{tt} - c^2 y_{xx} = 0, 0 < x < 1, t > 0,$$
$$y(0,t) = y(1,t) = 0$$
$$y(x,0) = 0, \quad 0 \le x \le 1$$
$$y_t(x,0) = x^2, \ 0 \le x \le 1$$

UNIT – IV

4. a) Prove that the solution of the Neumann problem is unique upto the addition of a constant. **10**

b) Show that the solution of the Dirichlet problem is stable. **10**

OR

c) Find the solution of the problem. **10**

$$\nabla^2 u = 0, \ -\infty < x < \infty, y > 0$$
$$u_y(x,0) = g(x), \ -\infty < x < \infty$$

Such that u is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$ and $\int_{-\infty}^{\infty} g(x)\,dx = 0$

d) Prove that the solution u(x, t) of the differential equation **10**

$$u_t - k u_{xx} = F(x,t), 0 < x < \ell, t > 0$$

Satisfying the initial condition.

u(x, 0) = f(x), $0 \le x \le \ell$

and the boundary condition $u(0,t) = u(\ell,t) = 0, t \ge 0$ is unique.

5. a) Find a complete integral of the p. d. e. **5**

$$z = px + qy + pq$$

b) Find analytic expression for the Monge cone at (x_0, y_0, z_0). **5**

c) Define: **5**

i) Analytic function. ii) Two dimensional Laplace’s equation .

d) Prove that the solution of Dirichlet problem, if exists is unique. **5**

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