M.Sc.- II (Mathematics) New CBCS Pattern Semester-IV

PSCMTH16 : Dynamical Systems

P. Pages : 2 | GUG/W/23/13767

Time : Three Hours | Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let $u:[0, \alpha] \to R$ be continuous & non negative. Suppose $C \geq 0,\ K \geq 0$ are such that $u(t) \leq C + \int_0^1 K\, u(s)\, dS$ for all $t \in [0, \alpha]$ then prove that $u(t) \leq C e^{kt}$ for all $t \in [0, \alpha]$. **10**

b) If $f: W \to E$ is locally Lipschitz & $A \subset W$ is a compact (Closed & bounded) set, then prove that $f \mid A$ is Lipschitz. **10**

OR

c) Prove that ϕ has the following property : $\phi_{s+t}(x) = \phi_s(\phi_t(x))$ in the sense that if one side of above equation is defined, so is the other, and they are equal. **10**

d) Find a Lipschitz constant on the region indicated $f(x) = x^{1/3}$, $-1 \leq x \leq 1$. **10**

UNIT – II

2. a) Let $\bar{x}$ be an isolated minimum of V. Then prove that $\bar{x}$ is an asymptotically stable equilibrium of the gradient system $x' = -\text{grad}\, V(x)$. **10**

b) Find equilibrium points of gradient system $f(z) = -\text{grad}\, V(z)$ where $V(x,\ y) = x^2 (x-1)^2 + y^2$ and $V: R^2 \to R$ be a function. **10**

OR

c) Prove that E* is isomorphic to E and thus has the same dimension. **10**

d) Let E be a real vector space with an inner product & let T be a self-adjoint operator on E. Then prove that the eigenvalues of T are real. **10**

UNIT – III

3. a) Let $y \in L_w(x) \cup L_\alpha(x)$. Then prove that the trajectory of y crosses any local section at not more than one point. **10**

b) Prove that **10**

i) If x and z are on the same trajectory, then $L_w(x) = L_w(z)$ similarly for α-limits.

ii) If D is a closed positively invariant set and $Z \in D$, then $L_w(Z) \subset D$, similarly for negatively invariant sets & α-limits.

OR

c) Let r be a closed orbit enclosing an open set U contained in the domain W of the dynamical system. Then prove that U contains an equilibrium. **10**

d) Prove that let S be a local section at O and suppose $\phi_{t_0}(z_0) = 0$. There is an open set $U \subset W$ containing τ_0 & a unique C^1 map $\tau : U \to R$ such that $\tau(z_0) = t_0$ and $\phi_{\tau(x)}(x) \in S$ for all $x \in U$. **10**

UNIT – IV

4. a) Let $g : S_0 \to S$ be a Poincare map for γ. Let $x \in S_0$ be such that $\lim\limits_{n\to\infty} g^n(x) = 0$. Then prove that $\lim\limits_{t\to\infty} d(\phi_t(x), \gamma) = 0$. **10**

b) Prove that let $\bar{x}$ be a fixed point of a discrete dynamical system $g : W \to E$. If the eigen values of $D g(\bar{x})$ are less than 1 in absolute value, $\bar{x}$ is asymptotically stable. **10**

OR

c) Assume E is normed. Let $\gamma > \left\| D + (x_0)^{-1} \right\|$ let $V \subset W$ be an open ball around x_0 such that $\left\| D + (y)^{-1} \right\| < \gamma$ and $\| D + (y) - D + (z) \| < \dfrac{1}{\gamma}$ for all $y, z \in V$. Then prove that $f \mid V$ is one-to-one. **10**

d) Let $W \subset R X E$ be open & $f, g : W \to E$ continuous. Suppose that for all $(t, x) \in W$, $| f(t, x) - g(t, x) | < \in$. Let K be a Lipschitz constant in x for f(t, x) If x(t), y(t) are solutions to $x' = f(t, x),\ y' = g(t, y)$ respectively, on some interval J, and $x(t_0) = y(t_0)$ then prove that $| x(t) - y(t) | \leq \dfrac{\in}{K} \left(\exp\left(K \mid t - t_0 \mid \right) - 1 \right)$. **10**

5. a) Define the flow of differential equation. **5**

b) Show that at an equilibrium of a gradient system, the eigenvalues are real. **5**

c) Define monotone sequences in planar dynamical systems. **5**

d) Explain structural stability. **5**

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