M.Sc. - II (Mathematics) New CBCS Pattern Semester-III

PSCMTH14C - (Optional) Paper-XIV : Graph Theory

P. Pages : 2 **GUG/W/23/13760**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let G be a acyclic graph with n vertices and k connected components. Then Prove that G has n-k edges. **10**

b) Prove that if G is a non-empty graph with at least two vertices. Then G is bipartite if and only if it has no odd cycles. **10**

OR

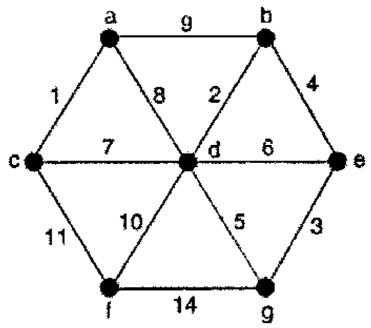
c) Let T be a tree with at least two vertices and if $P = u_0 u_1 \ldots u_n$ be a longest path in T, then prove that both u_0 and u_n have degree 1. **10**

d) Prove that if T is a tree with n-vertices then it has precisely n-1 edges. **10**

UNIT – II

2. a) Let G be a weighted connected graph in which the weights of the edges are all non-negative numbers. Let T be a subgraph of G obtained by Kruskal's algorithm. Then prove that T is a minimal spanning tree of graph G. **10**

b) Apply Kruskal's algorithm to the graph to find an optimal tree. **10**



OR

c) Prove that : A connected graph G is Euler if and only if the degree of every vertex is even. **10**

d) Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices. **10**

UNIT – III

3. a) Let G be a connected plane graph with n vertices, e edges and f faces. Let n*, e* and f* denote the number of vertices, edges and faces respectively of G* then prove that n* = f, e* = e, and f* = n. **10**

b) Let G be a plane graph without loops. If G has a Hamiltonian cycle C and α_i denotes the number of faces of degree I lying inside the cycle C and β_i denote the number of faces of degree I lying outside the cycle C, then prove that $\sum_i (i-2)(\alpha_i - \beta_i) = 0$. **10**

OR

c) Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian. **10**

d) Let G be a plane connected graph then prove that G is isomorphic to its double dual G**. **10**

UNIT – IV

4. a) Prove that a simple graph G is n-edge connected if and only if, given any pair of distinct vertices u and v of G, there are at least n internally disjoint paths from u to v. **10**

b) State and prove Max-Flow, Min-cut theorem. **10**

OR

c) State and prove first theorem of Digraph theory. **10**

d) Prove that: A tournament T is Hamiltonian if and only if it is strongly connected. **10**

5. a) Define: **5**
i) Adjacency matrix of a graph.
ii) Incidence matrix of a graph.

b) Let G be a connected graph with at least three vertices. Prove that if G has a bridge then G has a cut vertex. **5**

c) Let G_1 and G_2 be two plane graphs which are both redrawing's of the same planar graph G. Then prove that $f(G_1) = f(G_2)$. **5**

d) Define: **5**
i) Directed Hamiltonian path
ii) Directed Hamiltonian cycle.

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