M.Sc.(Mathematics) New CBCS Pattern Semester-III

PSCMTH13 - Paper-III : Mathematical Methods

P. Pages : 3 | **GUG/W/23/13757**

Time : Three Hours | Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Find the function whose cosine transform is **10**

$$\sqrt{\frac{2}{\pi}}\,\frac{\sin a\xi}{\xi}$$

b) Find Fourier sine transform of $f(x)=\dfrac{1}{x(x^2+a^2)}$. **10**

OR

c) Evaluate Fourier transform of $H(x+a)-H(x-a)$. **10**

d) Find Fourier sine & Fourier Cosine Transform of the function **10**

$$f(x)=\begin{cases}\sin x, & 0<x<a\\ 0, & x>a\end{cases}$$

UNIT – II

2. a) Let $f(x)$ be continuous and $f'(x)$ be sectionally continuous on the interval $0\le x\le a$, then prove that **10**

i) $\bar{f}_c\left[f'(x); x\to n\right]=(-1)^n f(a)-f(0)+\dfrac{n\pi}{a}\bar{f}_s(n), n\in Z^*$

ii) $\bar{f}_s\left[f'(x); x\to n\right]=\dfrac{-n\pi}{a}\bar{f}_c(n), n\in N$

b) Solve the wave equation **10**

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ 0\le x\le a, t>0$$

Satisfying the boundary conditions $u(0,t)=u(a,t)=0, t>0$ and the initial conditions

$u(x,0)=\dfrac{4b}{a^2}x(a-x),\ \dfrac{\partial u(x,0)}{\partial t}=0,\ 0\le x\le a$ to determine the displacement $u(x,t)$.

OR

c) The end points of a solid bounded by $x = 0$ and $x = \pi$ are maintained at temperatures $u(0,t) = 1$, $u(\pi,t) = 3$, where $u(x,t)$ represents its temperature at any point of it at any time t. Initially, the solid was held at 1 unit temperature with its surfaces were insulated. Find the temperature distribution $u(x,t)$ of the solid, given that $u_{xx}(x,t) = u_t(x,t)$. **10**

d) Solve the three dimensional Laplace Equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$. **10**

$0 \le x \le \pi,\ 0 \le y \le \pi,\ 0 \le z \le \pi$ with the boundary conditions

$V = V_0$, when $y = \pi; V = 0$, when $y = 0$ and

$V = 0$, when $x = 0, \pi$ and $V = 0$, when $z = 0, \pi$

UNIT – III

3. a) i) If Laplace transform of $f(t)$ is $\bar{f}(p)$, then prove that Laplace transform of $f(t-a)H(t-a)$ is $e^{-ap}\bar{f}(p)$. **10**

ii) If $L[f(t); t \to p] = \bar{f}(p)$, then prove that $L[f(at); t \to p] = \frac{1}{a}\bar{f}\left(\frac{p}{a}\right)$.

b) Define Laplace transform of the error function and Laplace transform of the error complementary function and evaluate Laplace transform of $E_r f\left(\sqrt{t}\right)$ and $L\left[E_r f_c\left(\sqrt{t}\right)\right]$. **10**

OR

c) Evaluate $L^{-1}\left[\frac{1}{p(p+1)^3}\right]$ **10**

d) Evaluate $L^{-1}\left[\frac{p}{(p^2+u)^3}\right]$ by using convolution theorem. **10**

UNIT – IV

4. a) If $f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases}$ Find Hankel transform of order x of $f(x)$. **10**

b) Solve the differential equation, **10**

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} = 0, r \ge 0, z \ge 0$$

Satisfying the conditions (i) $u \to \infty$ as $z \to \infty$ and as $r \to \infty$.

(ii) $u = f(r)$, on $z = 0$ $r \ge 0$.

OR

c) If $f^*(s)$ and $g^*(s)$ be Mellin Transforms of $f(x)$ and $g(x)$ respectively then prove that $M\left[f(x)g(x); x \to s\right] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f^*(z) \cdot g^*(s-z)\,dz$. **10**

d) Find Mellin inversion of $\overline{|(s)}$. **10**

5. a) If $F\left[f(x); x \to \xi\right] = F(\xi)$, then prove that $F\left[f(ax); x \to \xi\right] = \frac{1}{a}F\left(\frac{\xi}{a}\right)$. **5**

b) Let $f(x)$ & $f'(x)$ be continuous & $f''(x)$ be sectionally continuous in $0 \le x \le a$ then prove that $\bar{f}_c\left[f''(x); n\right] = -f'(0) + (-1)^n f'(a) - \frac{n^2\pi^2}{a^2}\bar{f}_c(n)$. **5**

c) Evaluate $L\left[\frac{1}{\sqrt{\pi t}}\right]$ **5**

d) Evaluate $H_1\left[\frac{e^{-ax}}{x};\ \xi\right]$ **5**

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