M.Sc.- II (Mathematics) New CBCS Pattern Semester-III

PSCMTH11 - Paper-I - Complex Analysis

P. Pages : 2 | GUG/W/23/13755

Time : Three Hours | * 7 3 4 1 * | Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that when a limit of a function f(z) exists at a point z_0, it is is unique. **10**

b) Suppose that $f(z) = u(x, y) + i\forall(x, y)$ and that $f'(z)$ exists at point $z_0 = x_0 + iy_0$. Prove that the first-order partial derivatives of u and ν must exist at (x_0, y_0), and must satisfy Cauchy-Riemann equations $u_x = v_y$, $u_y = -\nu_x$. **10**

OR

c) Suppose that a function f is analytic in some domain D which contains a segment of the x axis and whose lower half is the reflection of the upper half with respect to that axis. Then prove that $\overline{f(z)} = f(\bar{z})$ for each point z in the domain if and only if f(x) is real for each point x on the segment. **10**

d) If a function $f(z) = u(x, y) + i\nu(x, y)$ is analytic in a domain D, then prove that its component functions u and ν are harmonic in D. **10**

UNIT – II

2. a) Let C denote a contour of length L, and suppose that a function f(z) is piecewise continuous on C. Prove that if M is a nonnegative constant such that $|f(z)| \le m$ for all points z on C at which f (z) is defined, then $\left|\int_C f(z)\,dz\right| \le mL$. **10**

b) Let f be analytic everywhere inside and on a simple closed contour C, taken in the positive sense. If z_0 is any point interior to C, then prove that $f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)\,dz}{z - z_0}$ **10**

OR

c) Prove that if a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1 (z_1 \ne z_0)$, then it is absolutely convergent at each point z in the open disk.
$|z - z_0| < R_1$ where $R_1 = |z_1 - z_0|$. **10**

d) Prove that if a function f is entire and bounded in the complex plane, then f(z) is constant throughout the plane. **10**

UNIT – III

3. a) Let C be a simple closed contour, described in the positive sense. If a function f is analytic inside and on C except for a finite number of singular points $z_k\ (k=1, 2, ..., n)$ inside C, then. $\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} \underset{z=z_k}{\text{ReS}}\, f(z)$. **10**

b) Suppose that a function f is bounded and analytic in some deleted neighbourhood $0<|z-z_0|<\in$. Prove that if f is not analytic at z_0, then it has a removable singularity there. **10**

OR

c) Evaluate the improper integral. **10**

$$\int_0^\infty \frac{x\sin 2x}{x^2+3}dx.$$

d) Suppose that z_0 is an essential singularity of a function f, and let w_0 be any complex number. Then prove that, for any positive number $\in$, the inequality $|f(z)-w_0|<\in$ is satisfied at some point z in each deleted neighbourhood $0<|z-z_0|<\delta$ of z_0. **10**

UNIT - IV

4. a) Find a linear fractional transformation that maps the points $z_1=2,\ z_2=i$ and $z_3=-2$ on to the points $w_1=1,\ w_2=i$ and $w_3=-1$. **10**

b) Show that the transformation $w=\sin z$ is a one to one mapping of the sem-infintie strip. $-\pi/2\le x\le \pi/2,\ y\ge 0$ in the z plane onto the upper half $\nu\ge 0$ of the w plane. **10**

OR

c) Show that the mapping $w=1/z$ transforms circles and lines into circles and lines. **10**

d) Show that the image of the vertical strip $0\le x\le 1,\ y\ge 0$ under the mapping $w=z^2$ is a closed semiparabolic region. **10**

5. a) If a function f(z) is continuous and nonzero at a point z_0, then prove that $f(z)\ne 0$ throughout some neighbourhood of that point. **5**

b) Prove that a function f that is analytic throughout a simply connected domain D must have an antiderivative everywhere in D. **5**

c) Suppose that- **5**

i) Two functions p and q are analytic at a point z_0.

ii) $p(z_0)\ne 0$ and q has a zero of order m at z_0.

Then prove that the quotient p(z)/q(z) has a pole of order m at z_0.

d) Give a geometric description of the transformation $w=A(Z+B)$, where A and B are complex constants and $A\ne 0$. **5**

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