M.Sc.-I (Mathematics) (New CBCS Pattern) Semester - II

PSCMTHT10A - Optional Paper : Differential Geometry

P. Pages : 2 **GUG/S/23/13750**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If W is the angle between the parametric curves at the point of intersection then obtain $\tan W = \frac{H}{F}$ **10**

b) If $(\ell, m) \& (\ell', m')$ are the direction coefficients of two directions at a point P on the surface & θ is the angle between the two directions at P, then prove that **10**

i) $\cos\theta = E\,\ell\ell' + F(\ell m' + \ell' m) + Gmm'$

ii) $\sin\theta = H(\ell m' - \ell' m)$

OR

c) Show that the parameters an a surface can always be chosen so that the curves of the given family & the orthogonal trajectories become parametric curves. **10**

d) Show that the curves bisecting the angle between the parametric curves are given by $Edu^2 - Gdv^2 = 0$. **10**

UNIT – II

2. a) Prove that the curves of the family $\frac{v^3}{u^2} =$ constant are geodesics on a surface with the metric **10**

$v^2du^2 - 2uvdudv + 2u^2dv^2$, u > 0, v > 0

b) Prove that every helix on a cylinder is a geodesic and conversely. **10**

OR

c) If U & V are the intrinsic quantities of a surface at a point (u,v) then prove that **10**

$k_g = \frac{1}{N}\frac{v(s)}{u'}$.

d) Find the Gaussian curvature at a point (u,v) of the anchor ring. **10**

$r = ((b + a\cos u)\cos v, (b + a\cos u)\sin v, a\sin u)$,

$0 < u, v < 2\pi$

UNIT – III

3. a) Find L, M, N for the sphere $r = (a\cos u\ \cos v, a\cos u \sin v, a\sin u)$ where u is the latitude & v is the longitude. **10**

b) Find the principal directions & principal curvatures at a print on the surface **10**
$X = a(u+v),\ y = b(u-v),\ z = uv$

OR

c) Show that all points on a surface are umbilics. **10**

d) If K is the normal curvature in a direction making an angle Ψ with the principal direction V = constant then prove that $k = k_a \cos^2\Psi + k_b \sin^2\Psi$. Where $k_a\ \&\ k_b$ are principal curvatures at the point P on the surface. **10**

UNIT – IV

4. a) If $N_1 = \frac{\partial N}{\partial u}\ \&\ N_2 = \frac{\partial N}{\partial v}$ then prove that **10**

i) $N_1 = \frac{1}{H^2}\left[(FM - GL)r_1 + (FL - EM)r_2\right]$

ii) $N_2 = \frac{1}{H^2}\left[(FN - GM)r_1 + (FM - EN)r_2\right]$

b) If N is the surface normal then prove that $N_1 \text{ x } N_2 = \frac{LN - M^2}{H} N.$ **10**

OR

c) If $K_a\ \&\ K_b$ are the principal curvatures, then prove that **10**

$(k_a)_2 = \frac{1}{2}\frac{E_2}{E}(k_b - k_a)$ and

$(k_b)_1 = \frac{1}{2}\frac{G_1}{G}(k_a - k_b)$

d) If $\bar{k}$ and $\bar{\mu}$ are the Gaussian curvature & mean curvature of $\bar{S}$ then prove that **10**

$$\bar{k} = \frac{ke}{\left(1 + 2\mu a + ka^2\right)},\quad \bar{\mu} = \frac{(\mu + ak)e}{1 + 2\mu a + ka^2} \text{ Where } e = \pm 1$$

5. a) Obtain $ds^2 = Edu^2 + 2Fdudv + Gdv^2$ **5**

b) Show that the geodesics on a right circular cylinder are helices. **5**

c) Define **5**
i) Mean curvature μ
ii) Gaussian curvature K.

d) From the Weingarten equations, obtain $H[N, N_1, r_1] = EM - FL$. **5**

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