M.Sc.- I (Mathematics) New CBCS Pattern Semester-II

PSCMTH10A / PSCMTHT10A : Optional Paper : Differential Geometry

P. Pages : 2 **GUG/W/23/13750**

Time : Three Hours Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that the first fundamental form of a surface is a positive definite quadratic form in du, dv. **10**

b) If ω is the angle between the parametric curves at the point of intersection the prove that $T_{an} w = \frac{H}{F}$ **10**

OR

c) If (ℓ', m') are the direction coefficient of a line which makes an angle $\frac{\pi}{2}$ with then line whose direction coefficients are (ℓ, m) then prove that **10**

$$t' = \frac{-1}{H}(F\ell + Gm),\ m' = \frac{1}{H}(E\ell + Fm)$$

d) Find the orthogonal trajectories of the circle $r = a\cos\theta$. **10**

UNIT – II

2. a) If the arc length s is the parameter of the curve, then prove that geodesic equations are **10**

$$U = \frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right) - \frac{\partial T}{\partial u} = 0$$

$$V = \frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right) - \frac{\partial T}{\partial v} = 0$$

b) If the orthogonal trajectories of the curve v=constant are geodesics, then prove that $\left(\frac{H^2}{E}\right)$ is independent of u. **10**

OR

c) Find the Gaussian curvature at any point of a sphere with representation **10**
r = a (sin u cos v, sin u sin v, cos u)
where $0 < u < \pi$ and $0 \le v < 2\pi$.

d) Prove that in the geodesic polar form, The Gaussian curvature $k = -\frac{9_{11}}{g}$ where $g = \sqrt{G}$ of the surface. **10**

UNIT – III

3. a) Obtain $k_n = \dfrac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2fdudv + Gdv^2}$, if k_n is the normal curvature of the curve at a point on a surface and $L = N \cdot r_{11},\ M = N \cdot r_{12},\ N = N \cdot r_{22}$ & E, F, G are the first fundamental coefficients. **10**

b) Find the normal curvature of the right helicoid $r(u, v) = (u\cos v, u\sin v, cv)$ at a point on it. **10**

OR

c) Show that the principal curvatures are given by the roots of the equation **10**
$k^2\left(EG - F^2\right) - k(EN + GL - 2FM) + LN - M^2 = 0$

d) Show that a necessary & sufficient condition that the lines of curvature be the parametric curves is that F = 0, M = 0. **10**

UNIT – IV

4. a) If k_a & k_b are the principal curvatures, then prove that the Codazzi equations are **10**

$(k_a)_2 = \dfrac{1}{2}\dfrac{E_2}{E}(k_b - k_a)$ &

$(k_b)_1 = \dfrac{1}{2}\dfrac{G_1}{G}(k_a - k_b)$

b) State and prove Weingarten Equations. **10**

OR

c) Prove that the parallel surfaces of a minimal surface are surfaces for which $R_a + R_b =$ constant, where **10**

$R_a = \dfrac{1}{k_a}$ & $R_b = \dfrac{1}{k_b}$

d) Prove that from Weingarten equations **10**
$H[N,\ N_1,\ r_1] = EM - FL$ and
$H[N,\ N_1,\ r_2] = FM - GL$

5. a) Prove that if ds represents the element of area PQRS on the surface, ds = H du dv. **5**

b) If a geodesic on a surface of revolution cuts the meridian at a constant angle, then prove that the surface is a right cylinder. **5**

c) Prove that the straight lines on a surface are asymptotic lines. **5**

d) Show that when the lines of curvature are parametric curves, then the Weingarten equations are **5**

$N_1 = -\dfrac{L}{E}r_1$ & $N_2 = -\dfrac{N}{G}r_2$.

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