M.Sc. (Mathematics) (New CBCS Pattern) Semester - II

PSCMTH08 - Advanced Topics in Topology

P. Pages : 2 GUG/S/23/13748

Time : Three Hours * 2 4 8 4 * Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that a normal space is completely regular iff it is regular. **10**

b) Prove that every countably compact metric space is totally bounded. **10**

OR

c) Prove that a topological space X is completely normal iff every subspace of X is normal. **10**

d) Prove that every separable metric space is second axiom. **10**

UNIT – II

2. a) Prove that $\pi_\lambda X_\lambda$ is locally connected iff each space X_λ is locally connected and all but a finite number are connected. **10**

b) Prove that $X \times Y$ is connected iff X and Y are connected. **10**

OR

c) If X & Y are topological spaces, prove that the family of all sets of the form $V \times W$ with V open in X & W open in Y is a base for a topology for $X \times Y$ **10**

d) Prove that $\pi_\lambda X_\lambda$ is Hausdorff iff each space X_λ is Hausdorff . **10**

UNIT – III

3. a) If X is locally connected then prove that Y is locally connected with the quotient topology. **10**

b) Prove that for every open covering of a metric space, there is a locally finite open cover which refines it. **10**

OR

c) Prove that a subset G of Y is open in the quotient topology relative to $f : X \to Y$ iff $f^{-1}(G)$ is an open subset of X. **10**

d) Prove that every paracompact regular space is normal. **10**

UNIT – IV

4. a) If $(D, \geq)$ is a directed set and E is an eventual subset of D, then prove that E, with the restriction of $\geq$ is a directed set. Moreover, prove that a net $S : D \to X$ where X is a topological space, converges to x in X iff the restriction $S | E : E \to X$ converges to x in X. **10**

b) Prove that a topological space is compact iff every ultrafilter in it is convergent. **10**

OR

c) Prove that a topological space is Hausdorff iff limits of all nets in it are unique. **10**

d) Let $\{X_i : i \in I\}$ be a collection of nonempty spaces & let X be its topological product. Then prove that X is compact iff each X_i is so for $i \in I$. **10**

5. a) Prove that the family of all balls of points in a set X with metric d forms a base for a topology for X. **5**

b) Prove that the projections π_X & π_Y are continuous and open mappings. **5**

c) Define: **5**

i) Quotient topology

ii) σ - locally finite family

d) Define: **5**

i) Directed set

ii Net

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