M.Sc.(Mathematics) New CBCS Pattern Semester-II

PSCMTH08 : Advanced Topics in Topology

P. Pages : 2 GUG/W/23/13748

Time : Three Hours * 7 3 3 4 * Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that every metric space is completely normal. **10**

b) Prove that every Lindel of metric space is second axiom. **10**

OR

c) Prove that a normal space is completely regular iff it is regular. **10**

d) Prove that every metric space is a Hausdorff space. **10**

UNIT – II

2. a) Prove that the projections $\pi_x \& \pi_y$ are continuous and open mappings but not necessarily closed, & so the product topology is the smallest topology for which the projections are continuous. **10**

b) Prove that XXY is dense-in-itself iff at least one of the spaces X & Y is dense-in-itself. **10**

OR

c) Prove that $\pi_\lambda X_\lambda$ is connected iff each space X_λ is connected. **10**

d) Prove that $\pi_\lambda X_\lambda$ is compact iff each space X_λ is compact. **10**

UNIT – III

3. a) If F is a continuous, open mapping of the topological space X onto the topological space Y, then prove that the topology for Y must be the quotient topology. **10**

b) Prove that every paracompact regular space is normal. **10**

OR

c) If X is a regular paracompact space & Y is a regular 6-compact space, then prove that $X \times Y$ is paracompact. **10**

d) Prove that for every open covering of a metric space, there is a locally finite open cover which refines it. **10**

UNIT – IV

4. a) Let $S : D \to X$ be a net in a topological space and Let $x \in X$. Then prove that x is a cluster point of S iff there exists a subnet of 5 which converges to x in X. **10**

b) Let $\tau_1 \& \tau_2$ be topologies on a set X such that a net in X converges to a point w.r.t. τ_1 iff it does 50 w.r.t. τ_2. Then prove that $\tau_1 = \tau_2$. **10**

OR

c) Prove that every filter is contained in an ultrafilter. **10**

d) Prove that an ultrafilter converges to a point iff that point is a cluster point of it. **10**

5. a) Prove that every metric space is a first axiom space. **5**

b) If XxY is compact, then prove that X & Y are compact. **5**

c) Define **5**

i) Discrete family

ii) 6-discrete family.

d) Define **5**

i) Eventual subset

ii) Cofinal subset

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