M.Sc.(Mathematics) New CBCS Pattern Semester-II

PSCMTH07 : Lebesgue Measure Theory

P. Pages : 2 **GUG/W/23/13747**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all questions.
2. Each question carry equal marks.

UNIT – I

1. a) Show that the outer measure of an interval is its length. **10**

b) If $E_1 \& E_2$ are measurable then show that $E_1 \cup E_2$ is also measurable. **10**

OR

c) Show that the interval (a, ∞) is measurable. **10**

d) Let $\langle E_n \rangle$ be an infinite decreasing sequence of measurable sets i.e. a sequence with $E_{n+1} \subset E_n$ for each $n \& mE_1$ be finite then show that **10**

$$m\left(\bigcap_{i=1}^{\infty} Ei\right) = \lim_{n\to\infty} m\, E_n .$$

UNIT – II

2. a) Let $\phi \& \psi$ be simple functions which Vanish outside a set of finite measure then show that $\int (a\phi + b\psi) = a\int \phi + b\int \psi$. Also, if $\phi \geq \psi$ a.e. then show that $\int \phi \geq \int \psi$. **10**

b) State & prove the bounded convergence theorem. **10**

OR

c) State & prove the Fatou's lemma. **10**

d) Let f be a non negative function which is integrable over a set E then show that for $\in > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$ we have $\int_A f < \in$ **10**

UNIT – III

3. a) If f is of bounded variation on [a, b] then show that **10**

$$T_a^b = P_a^b + N_a^b \ \& \ f(b) - f(a) = P_a^b - N_a^b$$

b) State & prove the Vitali lemma. **10**

OR

c) If f is absolutely continuous on [a, b] & $f'(x)=0$ a.e. then show that f is constant. **10**

d) If f is bounded & measurable on [a, b] & $F(x)=\int_a^x f(t)\,dt+F(a)$ then show that $F'(x)=f(x)$ for almost all x in [a, b]. **10**

UNIT – IV

4. a) State & prove the holder inequality. **10**

b) Show that the LP spaces are complete **10**

OR

c) Let g be an integrable function on [0, 1] & suppose that there is a constant M such that $\left|\int fg\right| \le M\|f\|_p$ **10**

For all bounded measurable functions f then show that g is in L^p & $\|g\|_q \le M$.

d) State & prove Riesz representation theorem. **10**

5. a) If $m^*E=0$ then prove that E is measurable. **5**

b) Define the Riemann integral & Lebesgue integral. **5**

c) Define the convex functions. **5**

d) State the Minkowski inequality & its versions. **5**

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