M.Sc.(Mathematics) New CBCS Pattern Semester-II

PSCMTH06 : Field Theory

P. Pages : 2 **GUG/W/23/13746**

Time : Three Hours Max. Marks : 100

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Notes : 1. Solve all the **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) If R is a UFD, then prove that the factorization of any element in R as a finite product of irreducible factors is unique to within order and unit factors. **10**

b) Prove that every PID is a UFD, but a UFD is not necessarily a PID. **10**

OR

c) Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime. **10**

d) Prove that the product of two primitive polynomials in a UFD is primitive. **10**

UNIT – II

2. a) Let $F(x) \in \mathbb{Z}[x]$ be primitive. Then prove that F(x) is reducible over Q if & only if F(x) is reducible over $\mathbb{Z}$. **10**

b) Let $F \subseteq E \subseteq K$ be fields. If $[K:E] < \infty$ and $[E:F] < \infty$, then prove that **10**

i) $[K:F] < \infty$

ii) $[K:F] = [K:E]\,[E:F]$

OR

c) State and prove Eisenstein criterion. **10**

d) Let E be an algebraic extension of F, and let $\sigma: E \to E$ be an embedding of E into itself over F. Then prove that σ is onto and, hence an automorphism of E. **10**

UNIT – III

3. a) If the multiplicative group F* of non zero elements of a field F is cyclic, then prove that F is finite. **10**

b) If E is a finite separable extension of a field F, then prove that E is a simple extension of F. **10**

OR

c) Prove that the prime field of a field F is either isomorphic to Q or to ℤ /(P), P prime. **10**

d) Let $F \subset E \subset K$ be three fields such that E is a finite separable extension of F, and K is a finite separable extension of E. Then prove that K is a finite separable extension of F. **10**

UNIT – IV

4. a) State & prove fundamental theorem of algebra. **10**

b) If $f(x) \in F[x]$ has r distinct roots in its splitting field E over F, then prove that the Galois group G(E/F) of f(x) is a subgroup of the symmetric group S_r . **10**

OR

c) Prove that the Galois group of $x^3 - 2 \in Q[x]$ is the group of symmetries of the triangle. **10**

d) Prove that the Galois group of $x^4 - 2 \in Q[x]$ is the octic group (= group of symmetries of a square). **10**

5. a) Define : **5**

i) Unique Factorization Domain &

ii) Euclidean domain

b) Show that $x^3 + 3x + 2 \in$ ℤ/(7) [x] is irreducible over the field ℤ/(7). **5**

c) Define : **5**

i) Splitting field &

ii) Separable polynomial.

d) Let $G = G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$. Then find |G|. **5**

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