M.Sc.- I (Mathematics) New CBCS Pattern Semester-I

PSCMTH05D - Number Theory

P. Pages : 2

GUG/W/23/13744

Time : Three Hours

* 7 3 3 0 *

Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) State and prove Wilson's theorem. **10**

b) Find the least positive integer x such that $x \equiv 5$ (mod 7), $x \equiv 7$ (mod 11), and $x \equiv 3$ (mod 13). **10**

OR

c) Show that 1387 is composite. **10**

d) Let q be a prime factor of $a^2 + b^2$. If $q \equiv 3$ (mod 4) then prove that $q \mid a$ and $q \mid b$. **10**

UNIT – II

2. a) Show that the congruence $f(x) \equiv 0$ (mod p) of degree n has at most n solutions. **10**

b) Solve $x^2 + x + 7$ (mod 81). **10**

OR

c) If p is a prime and $(a, p) = 1$, then prove that the congruence $x^n \equiv a$ (mod p) has $(n, p-1)$ solutions or no solution according as $a^{(p-1)/(n,p-1)} \equiv 1$ (mod p). **10**

d) Show that the congruence $x^5 \equiv 6$ (mod 101) has 5 solutions. **10**

UNIT – III

3. a) State and prove Gauss Lemma. **10**

b) State and prove the Gaussian reciprocity law. **10**

OR

c) Derive de Polignac's formula. **10**

d) Let x and y be real numbers. Then prove that **10**

1) $[x] \le x < [x] + 1, x - 1 < [x] \le x, 0 \le x - [x] < 1$

2) $[x] = \sum_{1 \le i \le x} 1$ if $x \ge 0$.

3) $[x+m] = [x] + m$ if m is an integer.

4) $[x] + [y] \le [x+y] \le [x] + [y] + 1$

UNIT – IV

4. a) Prove that for every positive integer n, $\sum_{d \backslash n} \phi(d) = n$ **10**

b) If $f(n) = \sum_{d \backslash n} \mu(d) F(n/d)$ for every positive integer n, then prove that $F(n) = \sum_{d \backslash n} f(d)$. **10**

OR

c) Find all solutions of $999x - 49y = 5000$. **10**

d) Show that the equation $y^2 = x^3 + 7$ has no solution in integers. **10**

5. a) Let f denote a polynomial with integral coefficients. If $a \equiv b$ (mod m) then prove that $f(a) \equiv f(b)$ (mod m). **5**

b) If $d \mid (p-1)$, then show that $x^d \equiv 1$ (mod p) has d solutions. **5**

c) Prove that 3 is a quadratic residue of 13, but a quadratic nonresidue of 7. **5**

d) Show that the equation $15x^2 - 7y^2 = 9$ has no solution in integers. **5**

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