M.Sc. - I (Mathematics) New CBCS Pattern Semester-I

PSCMTH05(A) - Numerical Analysis (Optional Paper)

P. Pages : 2 **GUG/W/23/13741**

Time : Three Hours * 7 3 2 7 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each questions carries equal marks.

UNIT – I

1. a) Let $f(x), f'(x), f''(x)$ are continuous for all value of x in some interval containing α and assume $f'(\alpha) \neq 0$ then prove that if the initial guesses x_0 and x_1 are chosen sufficiently close to α the iterates x_n of $x_{n+1} = x_n - f(x_n) = \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}, n \geq 1$ will converge to α. The order of convergence will be $P = (1+\sqrt{5})/2 \simeq 1.62$. **10**

b) Apply the Newton's method for the function **10**

$$f(x) = \sqrt{x},\ x \geq 0$$
$$= -\sqrt{-x},\ x < 0$$

with root $\alpha = 0$. What is the behavior of the iterates? Do they converge, and if so at what rate?

OR

c) Assumes $f(x), f'(x), f''(x)$ are continuous for all x in some neighbourhood of α and assume $f(\alpha) = 0$ sufficiently close to α the iterates $x_n, n \geq 0$ of $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, will converge to α. **10**

d) Discuss Muller's method for finding roots of a polynomial. Discuss why Muller's method is better than the secant method. **10**

UNIT – II

2. a) Prove that for $k \geq 0$ **10**

$f[x_0, x_1, ...x_k] = \frac{1}{k!n^k} \Delta^k f_0$ where $f_0 = f(x_0)$ & $f_i = f(x_i)$.

b) Define n^{th} order of Newton's divided difference of a function f namely $f[x_0, ...x_n]$ show that $f[x_0, ...x_n] = \frac{f[x_1, ...x_n] - f[x_0, ...x_{n-1}]}{x_n - x_0}$. **10**

OR

c) For the basis functions $l_{j,n}(x)$ given by $l_i(x)=\prod_{j\neq i}\left(\frac{x-x_j}{x_i-x_j}\right)$, $i=1,2,.....,n$. Then prove that for $n\geq 1, \sum_{j=0}^{n} l_{j,n}(x)=1$ for all x. **10**

d) For any two functions f and g, for any constant α and β prove that $\Delta^r(\alpha f(x)+\beta g(x))=\alpha\Delta^r f(x)+\beta\Delta^r g(x)$ for $r\geq 0$. **10**

UNIT-III

3. a) Let $f(x)$ be continuous for $a\leq x\leq b$ and Let $\in>0$ then prove that there is a polynomial $p(x)$ for which **10**

$$|f(x)-p(x)|\leq\in,\ a\leq x\leq b$$

b) Prove that for $f, g\in c[a,b]$ **10**

$$|(f,g)|\leq\|f\|_2\cdot\|g\|_2$$

OR

c) Find the linear least square approximation of the function $f(x)=e^x -1\leq x\leq 1$. **10**

d) To obtain a minimax polynomial approximation $q_1^*(x)$ for the function $f(x)=e^x$ on the interval $[-1, 1]$. **10**

UNIT – IV

4. a) Obtain the expression for Peano-Kernel error formula. **10**

b) Obtain the composite trapezoidal rule with error. Find the expression for the asymptotic error. **10**

OR

c) Obtain the formula for the simple Simpson's rule of integration, obtain error estimate. **10**

d) Derive Newton – cotes integration formula for n = 1. **10**

5. a) Show that, if $g(x)$ be continuous for $a\leq x\leq b$ and assume that $a\leq g(x)\leq b$ for every $a\leq x\leq b$. Then $x=g(x)$ has at least one solution in $[a,b]$. **5**

b) Obtain the expression for $p_1(x)$ by Lagrange interpolation. **5**

c) For $f, g\in c[a,b]$. Then prove that $\|f+g\|_2\leq\|f\|_2+\|g\|_2$. **5**

d) Obtain Simple Trapezoidal Rule. **5**

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