M.Sc. - I (Mathematics) New CBCS Pattern Semester-I

PSCMTH03 - Topology-I

P. Pages : 2 **GUG/W/23/13739**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) If $A \lesssim B$ and $B \lesssim A$, then prove that A ~ B. **10**

b) Prove that every infinite set contains a denumerable subset. **10**

OR

c) Prove that the set of all real numbers is uncountable. **10**

d) Prove that $2^{N_o} = \subset$ where N denotes the Hebrew alphabet aleph. **10**

UNIT – II

2. a) Let x = {a, b, c} & let $\tau = \{\phi, \{a\}, \{a, b\}, x\}$. Then find $d(\{a\})$ & $d(\{c\})$. **10**

b) For any set E in a topological space, prove that $C(E) = E \cup d(E)$. **10**

OR

c) For any set E in a topological space (x, τ), prove that $i(E) = \left[C\left(E^C\right)\right]^C$ **10**

d) Prove that A set is a closed subset of a topological space iff its complement is an open subset of the space. **10**

UNIT – III

3. a) Prove that the union E of any family $\{C_\lambda\}$ of connected sets having a nonempty intersection is a connected set. **10**

b) If f is a continuous mapping of (x, τ) into (x^*, τ^*), then prove that f maps every connected subset of x onto a connected subset of x* **10**

OR

c) Prove that a compact subset of a topological space is countably compact. **10**

d) If f is a one-to-one continuous mapping of (x, τ) into (x^*, τ^*), then prove that f maps every dense-in-itself subset of x onto a dense-in-it self subset of x*. **10**

UNIT – IV

4. a) Prove that In a T_1-space X, a point x is a limit point of a set E iff every open set containing x contains an infinite number of distinct points of E. **10**

b) Prove that every compact Hausdorff space is normal. **10**

OR

c) Prove that a topological space X satisfying the first axiom of countability is a Hausdorff space iff every convergent sequence has a unique limit. **10**

d) Prove that a topological space X is a T_0 – space iff the closures of distinct points are distinct. **10**

5. a) Define & give an example of equipotent sets & countable sets. **5**

b) Define a topology for a set X & give an example of a topology for some set X. **5**

c) Define (i) separation (ii) compact set (iii) Homeomorphism. **5**

d) Define (i) T_0 -space (ii) T_1 - space iii) T_2 - space. **5**

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