B.Sc.- III CBCS Pattern Semester-VI

021B - Mathematics Paper-III (DSE-VII) : Linear Programming and Transportation Problems

P. Pages : 4 **GUG/W/23/13364**

Time : Three Hours *6760* Max. Marks : 60

Notes : 1. Solve all **five** question.
2. All question carry equal marks.

UNIT – I

1. a) Find the initial feasible solution of **6**

$3x_1 + 4x_2 - x_3 \le 13$

$-x_1 + 5x_2 \le -4$

$x_1 \ge 0$, x_2 unrestricted in sign.

b) Put the following program in matrix standard form. **6**

$$\begin{aligned} \text{Minimize : } Z &= x_1 + 2x_2 + 3x_3 \\ \text{Subject to : } & 3x_1 + 4x_3 \le 5 \\ & 5x_1 + x_2 + 6x_3 = 7 \\ & 8x_1 + 9x_3 \ge 2 \\ \text{with : } & x_1, x_2, x_3 \ge 0 \end{aligned}$$

OR

c) Solve the following L. P. P graphically **6**

$$\begin{aligned} \text{Maximize : } Z &= 10x + 30y \\ \text{Subject to : } & x + 2y \le 20 \\ & x + 5y \le 35,\ x + 4y \le 48 \\ \text{with } & x, y \ge 0 \end{aligned}$$

d) Determine whether the set $\left\{[1,1,3,1]^T, [1,2,1,1]^T, [1,0,0,1]^T\right\}$ is linearly independent. **6**

UNIT – II

2. a) Solve the following L. P. P. by simplex method. **6**

$$\begin{aligned} \text{maximize : } Z &= 3x_1 + 4x_2 \\ \text{Subject to: } & 2x_1 + x_2 \le 6 \\ & 2x_1 + 3x_2 \le 9 \\ \text{with } & : x_1,\ x_2 \ge 0 \end{aligned}$$

b) Solve by two-phase method the following LPP. **6**

Minimize : $Z = 6x_1 + 3x_2 + 4x_3$

Subject to: $x_1 + 6x_2 + x_3 = 10$

$2x_1 + 3x_2 + x_3 = 15$

with : $x_1, x_2, x_3 \geq 0$

OR

c) Solve the following LPP by big M. method **6**

Maximize : $Z = 4x_1 + 5x_2 - 3x_3$

Subject to : $x_1 + x_2 + x_3 = 10$

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 + x_3 \leq 30$

with : $x_1, x_2, x_3 \geq 0$

d) Find the dual of the following LPP. **6**

Minimize : $Z = 3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5$

Subject to: $2x_1 + 5x_2 + x_4 + x_5 \geq 6$

$4x_2 - 2x_3 + 2x_4 + 3x_5 \geq 5$

$x_1 - 6x_2 + 3x_3 + 7x_4 + 5x_5 \leq 7$

with : $x_1, x_2, x_3, x_4, x_5 \geq 0$

UNIT – III

3. a) Use north-west corner rule to determine an initial basic feasible solution to the following transportation problem. **6**

	D_1	D_2	D_3	Supply
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Demand	7	9	18	

b) Explain the steps of determining an initial basic feasible solution to the transportation problem. By Vogel's Approximation method. **6**

OR

c) Determine an initial basic feasible solution to the following transportation problem using Vogel's approximation method. **6**

	A	B	C	D	Supply
I	1	2	1	4	30
II	3	3	2	1	50
III	4	2	5	9	20
Demand	20	40	30	10	

d) A plastics manufacture has 1200 boxes of transparent wrap in stock at one factory and another 1000 boxes at its second factory the manufacturer has ordered for this product from their different retailers, in quantities of 1000, 700 and 500 boxes respectively. The unit shipping costs (in cents per box) from the factories to retailers are as fallows. **6**

	Retailer 1	Retailer 2	Retailer 3
Factory 1	14	13	11
Factory 2	13	13	12

Determine minimum cost shipping schedule for satisfying all demands form current inventory.

UNIT – IV

4. a) Find all local and global optima for $f(x) = x + x^{-1}$ on $(0, \infty)$. **6**

b) Use Golden-mean search to approximate the location of the maximum of $f(x) = x(5\pi - x)$ on [0,20] to within $\in = 1$. **6**

OR

c) Prove that in an assignment problem, if we add (or subtract) a constant to every element of any row (or column) of the cost matrix $\left[C_{ij}\right]$ then an assignment that minimizes the total cost on one matrix will also minimize the total cost on the other matrix. **6**

d) A car hire company has one car at each of fine depots a, b, c, d and e. A customer requires a car in each town, namely.
A, B, C, D and E. Distance (in kms) between depots (origins) and towns (destinations) are given in the following distance matrix: **6**

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should cars be assigned to customers so as to minimize the distance travelled?

5. Solve **any six**

a) Write general form of linear programming problem. **2**

b) Define surplus variables. **2**

c) In simplex method when solution under test is not optimal. **2**

d) Write dual of the programme **2**

Maximize : $Z = \bar{C}^{T} \bar{X}$

Subject to : $A\bar{X} \le \bar{B}$

with : $\bar{X} \ge 0$

e) Define optimal solution to transportation problem. **2**

f) Define basic feasible solution to transportation problem. **2**

g) Define Global maximum. **2**

h) Show that $f(x) = x^3 - 6x^2 + 9x + 6$ is strictly concave on $(-\infty, 2)$ and strictly convex on $(2, \infty)$. **2**

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