# B.Sc.-III (CBCS Pattern) Semester - VI <br> <br> 021B - DSE-VII : Mathematics-III <br> <br> 021B - DSE-VII : Mathematics-III <br> <br> Linear Programming and Transportation Problems 

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P. Pages: 3

GUG/S/23/13364
Time : Three Hours

Notes: 1. Solve all five questions.
2. All questions carry equal marks.

UNIT - I

1. a) Find an initial feasible solution of the following.
$2 x_{1}+x_{3} \geq 5$
$5 x_{1}-2 x_{3} \geq-3$
$3 x_{1}+x_{2}-7 x_{3}=16$
b) Put the following program in matrix standard form

Maximize: $\mathrm{Z}=4 \mathrm{x}_{1}+2 \mathrm{x}_{2}$
Subject to: $3 \mathrm{x}_{1}+\mathrm{x}_{2} \geq 27$

$$
x_{1}+x_{2} \geq 21
$$

$$
x_{1}+2 x_{2} \geq 30
$$

with: $x_{1}, x_{2} \geq 0$

## OR

c) Solve the L. P. P. graphically

Maximize: $\mathrm{Z}=3 \mathrm{x}_{1}+2 \mathrm{x}_{2}$
Subject to: $\mathrm{x}_{1}+\mathrm{x}_{2} \leq 20$

$$
\begin{gathered}
\mathrm{x}_{1} \leq 15 \\
\mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 45 \\
-3 \mathrm{x}_{1}+5 \mathrm{x}_{2} \leq 60
\end{gathered}
$$

with: $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
d) Determine whether the set $\left\{[-1,2,1]^{\mathrm{T}},[3,0,-1]^{\mathrm{T}},[-5,4,3]^{\mathrm{T}}\right\}$ is linearly dependent
2. a) Solve following L. P. P. by simplex method.

Maximize: $\mathrm{Z}=\mathrm{x}_{1}+\mathrm{x}_{2}$
Subject to: $\mathrm{x}_{1}+5 \mathrm{x}_{2} \leq 5$

$$
2 x_{1}+x_{2} \leq 4
$$

with: $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
b) Solve the following LPP by two phase method

Maximize: $Z=80 x_{1}+60 x_{2}$
Subject to: $0.20 \mathrm{x}_{1}+0.32 \mathrm{x}_{2} \leq 0.25$

$$
\mathrm{x}_{1}+\mathrm{x}_{2}=1
$$

with: $x_{1}, x_{2} \geq 0$
c) Solve the following LPP by Big-M method.

Maximize: $\mathrm{Z}=3 \mathrm{x}_{1}+2.5 \mathrm{x}_{2}$
Subject to: $x_{1}+2 x_{2} \geq 20$

$$
3 x_{1}+2 x_{2} \geq 50
$$

with: $x_{1}, x_{2} \geq 0$
d) Determine the dual of L.P.P.

Maximize: $Z=6 x_{1}+5 x_{2}-7 x_{3}$
Subject to: $7 \mathrm{x}_{1}+11 \mathrm{x}_{2}+3 \mathrm{x}_{3} \leq 25$

$$
\begin{gathered}
2 x_{1}+8 x_{2}+6 x_{3} \leq 30 \\
6 x_{1}+x_{2}+7 x_{3} \leq 35
\end{gathered}
$$

with : $x_{1}, x_{2}, x_{3} \geq 0$

## UNIT - III

3. a) Use the north-west corner rule to obtain an initial basic feasible solution to the transportation problem.

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0_{1}$ | 19 | 30 | 50 | 10 | 7 |
| $0_{2}$ | 70 | 30 | 40 | 60 | 9 |
| $0_{3}$ | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 |  |

b) Explain the steps of determining initial basic feasible solution to transportation problem by Vogel's Approximation method.

## OR

c) Find the initial basic feasible solution of the transportation problem using least cost entry method.

d) Determine the optimal solution of the following transportation problem using Vogel's approximation method for initial basic solution.

|  | $\mathrm{W}_{1}$ |  | $\mathrm{~W}_{2}$ | $\mathrm{~W}_{3}$ | $\mathrm{~W}_{4}$ |
| ---: | :---: | :---: | :---: | :---: | :---: |
| Capacity |  |  |  |  |  |
|  | $\mathrm{F}_{1}$ | 11 | 20 | 7 | 8 |
| 50 |  |  |  |  |  |
| $\mathrm{~F}_{2}$ | 21 | 16 | 10 | 12 | 40 |
|  | 212 | 18 | 9 | 70 |  |
| $\mathrm{~F}_{3}$ | 8 | 12 | 18 |  |  |
| Requirement | 30 | 25 | 35 | 40 |  |

## UNIT - IV

4. a) Maximize: $\mathrm{Z}=x(5 \pi-x)$ on $[0,20]$.
b) Use three-points search to approximate the location of the maximum of $\mathrm{f}(\mathrm{x})=\mathrm{x}(5 \pi-\mathrm{x})$ on $[0,20]$ to within $\in=1$

## OR

c) Prove that optimum assignment schedule remains unaltered if we add (substract) a constant to (from) all the elements of the row or column of the assignment cost matrix.
d) HMT Ltd. decides to make four sub assemblies through four contractors. Each contractor is to receive only one sub assembly. The cost of each sub assembly is determined by the bids submitted by each contractor and is shown in the following table in hundreds of rupees. Assign the different sub assemblies to the contractors to minimize the total cost.

> Contractor

Sub assembly

| 1 | 2 | 3 | 4 |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 15 | 13 | 14 | 17 |
| 2 | 11 | 12 | 15 | 13 |
| 3 | 13 | 12 | 10 | 11 |
|  | 15 | 17 | 14 | 16 |
|  |  |  |  |  |

## 5. Solve any six.

a) Define Optimization problem.
b) Transform the constraint $4 \mathrm{x}_{1}+5 \mathrm{x}_{2}+3 \mathrm{x}_{3} \leq 1800$ into an equation using slacker surplus variables.
c) In simples method when solution under test is optimal.
d) Write dual of the programme.

Minimize: $Z=\overline{\mathrm{C}}^{\mathrm{T}} \overline{\mathrm{X}}$
Subject to: $\mathrm{A} \overline{\mathrm{X}} \leq \overline{\mathrm{B}}$
with : $\overline{\mathrm{X}} \geq 0$
e) Define basic feasible solution to transportation problem.
f) Write mathematical formulation of transportation problem.
g) If for an assignment $\mathrm{c}_{\mathrm{ij}} \geq 0$, then prove that an assignment schedule $\left(\mathrm{x}_{\mathrm{ij}}\right)$ which satisfies $\Sigma \Sigma \mathrm{x}_{\mathrm{ij}} \mathrm{c}_{\mathrm{ij}}=0$ must be optimal.
h) Define Convex functions.

