B.Sc-III. (CBCS Pattern) Semester - VI

021C - DSE-V : Mathematics-I : Numerical Methods

P. Pages : 3 **GUG/S/23/13363**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Use the Bisection method to find the negative root of $x^3 - 4x + 8 = 0$ to the four decimal places. **6**

b) Prove that the rate of convergence of the N-R method is 2 and is given by **6**

$$|e_{n+1}| = \left| -\frac{f''(r)}{2f'(r)} \right| |e_n|^2$$

Where r is the exact root of the equation $f(x) = 0$

OR

c) Solve the system of equations **6**

$0x_1 + 2x_2 - 3x_3 = 1, 3x_1 - x_2 + x_3 = 8, \; 2x_1 + x_2 - 2x_3 = 6$

by the Gauss elimination method with partial pivoting.

d) Obtained the triangular factorization of the matrix. **6**

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

UNIT – II

2. a) Let $y = f(x)$ be a polynomial of degree three. The following data gives entries y_0 to y_3: **6**

x	0	1	2	3	4	5
y	1	-1	3	25	-	-

Find the next two entries i. e. y_4 and y_5

b) Show that **6**

$$\mu\delta = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$$

OR

c) Using Newton-Gregory forward interpolation formula, estimate the value of sin52 from the following data **6**

x	45	50	55	60
y= sinx	0.7071	0.7660	0.8192	0.8660

Where the angles x are measured in degrees.

d) Using Lagrange interpolation formula, find the missing value from the following data: **6**

x	0	1	3	4	7
y	2	-1	-1	-	23

UNIT – III

3. a) Find the first derivative of the function $f(x)$ at $x=1$ from the give data: **6**

x	0	1	2	3	4	5
y	2	9	28	65	126	217

b) The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. **6**

t	0	5	10	15	20
v	0	10	70	180	340

Find the acceleration at t = 0.

OR

c) Find $y'(3)$ from the Lagrange interpolation formula for the function given by the values: **6**

x	3	5	7	9
y	5	8	12	17

d) Find the maxima and minima of the function $y=f(x)$ specified by the values: **6**

x:	-1	0	1	3	4
y:f(x)	-4	3	-4	12	131

UNIT – IV

4. a) Derive the trapezoidal rule from Lagrange form of Newton-Cotes formula. **6**

b) Evaluate the integral $\int_0^2 e^{x^2}\,dx$ by Simpsons one-third rule. **6**

OR

c) Evaluate the integral $\int_0^3 \frac{dx}{1+x^3}$ by Simpson three-eighth quadrature formula. **6**

d) Prove that the trapezoidal rule has degree of precision one. **6**

5. Solve **any six**.

a) Show that the Newton-Raphson iteration for determining a k th root of A is **2**

$$x_{n+1}=\frac{1}{k}\left[(k-1)x_n+\frac{A}{x_n^{k-1}}\right], n=0,1,2...$$

b) Define upper and lower triangular matrix. **2**

c) Define a factorial polynomial. **2**

d) Prove that $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$. **2**

e) Write the special Newton backward formula for first derivatives at tabular Points near $x = x_n$. **2**

f) Write the Newton divided difference formula for second derivatives. **2**

g) Evaluate the integral $\int_0^6 \frac{dx}{1+x}$ by the trapezoidal rule. **2**

h) Define an error constant. **2**

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