B.Sc.-III (CBCS Pattern) Semester - VI

021C - Mathematics Paper-IV - DSE-VIII : Special Relativity-II

P. Pages : 2 **GUG/S/23/13362**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve all the questions.
2. All questions carry equal marks.

UNIT – I

1. a) If $f = a_{rs} x^r x^s$ then show that: **6**

$$\frac{\partial f}{\partial x^r} = \left(a_{rs} + a_{sr}\right) x^s \ \& \ \frac{\partial^2 f}{\partial x^r \partial x^s} = a_{rs} + a_{sr}$$

b) A covariant vector has component $2x - z, x^2 y, yz$ in rectangular coordinates. Find its covariant components in cylindrical coordinates. **6**

OR

c) Define the inner product. If A^m, B_{nrs} are tensor then show that $A^m B_{mrs}$ is also a tensor. **6**

d) Show that δ^r_s is a mixed tensor of order two. **6**

Let A^{pq}_{rst} be a tensor. Choosing p = t, q = s show that A^{pq}_{rqp} is also a tensor. What is its rank?

UNIT – II

2. a) Show that $\Gamma^m_{mn} = \left(\log\sqrt{g}\right)_{,n}$ for $g < 0$. **6**

b) Find the nonvanishing components of Christoffel symbols of second kind for **6**
$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$

OR

c) Show that under a linear transformation of a coordinate system **6**
$x^m = a^m_n x'^n + b^m, a^m_n, b^m$ are constants, the Christoffel symbols are tensors.

d) Show that the covariant derivative of a scalar is its partial derivative. **6**

UNIT – III

3. a) Derive the expression for force in the transverse & longitudinal mass. **6**

b) Obtain the mass energy equivalence $E = mc^2$. **6**

OR

c) Prove that the four velocity, in component form can be expressed as **6**

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}} \right).$$

Where $\bar{u} = (u_x, u_y, u_z)$ is ordinary 3 dimensional velocity of the particle.

d) Show that $P^2 - E^2/C^2$ is an invariant whose numerical value is $m_0^2 c^2$. **6**

UNIT – IV

4. a) Write the expression for the scalar & vector potential, Express these equation in component form. **6**

b) Show that the Hamiltonian for a charged particle moving in an electromagnetic fields is **6**

$$H = \left[m_0^2 C^4 + C^2 \left(P - \frac{e}{C} A \right)^2 \right]^{\frac{1}{2}} + e\phi .$$

OR

c) Obtain the matrix f_{ij} in electromagnetic field tensor. **6**

d) Show that **6**

i) $Ey' = \alpha \left(Ey - \frac{\nu}{C} Hz \right)$ & ii) $Ez' = \alpha \left(Ez + \frac{\nu}{C} Hy \right)$

5. Solve **any six.**

a) Define Kronecker delta. **2**

b) Define symmetric & Skew symmetric tensor. **2**

c) Show that $[mn, r] = [nm, r]$. **2**

d) Show that $g_{mn}, r = 0$. **2**

e) Prove that $g_{ij} u^i u^j = 1$. **2**

f) Show that $\frac{dE}{dp} = u$ **2**

g) Write the component form of $\text{cur}\ell \bar{H} = \frac{1}{C} \frac{\partial \bar{E}}{\partial t}$. **2**

h) Show that $E^{-1} = \bar{E}$. **2**

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