B.Sc. (III) (CBCS Pattern) Semester - VI

021A - Mathematics-II - DSE-VI : Complex Analysis and Vector Calculus

P. Pages : 2 **GUG/S/23/13361**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Obtain the Cauchy - Riemann equations in polar form. **6**

b) Show that $w = e^{\overline{z}}$ is not analytic for any z. **6**

OR

c) Show that $u = x^3 - 3xy^2$ is harmonic & find the corresponding analytic function. **6**

d) Prove that every bilinear transformation with a single noninfinite fixed point α can be put in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + k$, $k \to$ constant **6**

UNIT – II

2. a) Evaluate $\int_c \left(z - z^2\right) dz$, where c is the upper half of the circle $|z| = 1$. **6**

b) If a function $f(z)$ is analytic in a simply connected domain D then show that $\int_c f(z)\,dz = 0$, for every simple closed curve C in D **6**

OR

c) Evaluate $\int_c \frac{15z+9}{z\left(z^2-9\right)} dz$, where c is the circle $|z-1| = 3$ **6**

d) Using Cauchy's formula evaluate $\int_c \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$ **6**

UNIT – III

3. a) If $\overline{A} = \left(2x^2y - x^4\right)\overline{i} + \left(e^{xy} - y\sin x\right)\overline{j} + x^2 \cos y\overline{k}$ then show that $\frac{\partial^2 \overline{A}}{\partial y \partial x} = \frac{\partial^2 \overline{A}}{\partial x \partial y}$ **6**

b) Prove that : **6**

i) $\overline{a} \cdot \nabla \overline{r} = \overline{a}$

ii) $\nabla \phi = \frac{-\overline{r}}{r^3}$ for $\phi = \frac{1}{r}$

OR

c) Evaluate $\int_c \overline{F}\cdot d\overline{r}$ from (0,0,0) to (1,1,1) along the straight line joining (0,0,0) & (1,1,1) when $\overline{F}=\left(3x^2+6y\right)\overline{i}-14yz\overline{j}+20xz^2\overline{k}$. **6**

d) If $\overline{F}=\left(2x+y^2\right)\overline{i}+\left(3y-4x\right)\overline{j}$, evaluate $\int_c \overline{F}\cdot d\overline{r}$ around the path, parabolic are $y=x^2$ joining (0,0) to (1, 1) **6**

UNIT – IV

4. a) Verify the Green's theorem in the plane $\int_c \left(xy+y^2\right)dx+x^2dy$ where C is the closed curve bounded by $y=x\ \&\ y=x^2$ **6**

b) Show that $\iint\limits_s \left(ax\overline{i}+by\overline{j}+cz\overline{k}\right)\cdot n\,ds=\frac{4}{3}\pi\left(a+b+c\right)$ where S is the surface of the sphere $x^2+y^2+z^2=1$ **6**

OR

c) Apply Stoke's theorem to evaluate $\oint\limits_c \left(ydx+zdy+xdz\right)$, where C is the curve of intersection of $x^2+y^2+z^2=a^2\ \&\ x+z=a$ **6**

d) State & prove the divergence theorem. **6**

5. Solve **any six**.

a) Show that $f(z)=xy+iy$ is not analytic **2**

b) Define harmonic & conjugate functions. **2**

c) Prove that $\int_c \frac{dz}{z-a}=2\pi i,\ \text{where}\ C:|z-a|=r$ **2**

d) State Cauchy's integral formula. **2**

e) If f & g are irrotational, show that $\overline{f}\times\overline{g}$ is solenoidal. **2**

f) Define the divergence and curl of vector. **2**

g) State the Green's theorem. **2**

h) State Stokes theorem. **2**

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