B.Sc.- III CBCS Pattern Semester-VI

021A - (DSE-VI) - Mathematics-II : Complex Analysis and Vector Calculus

P. Pages : 2 **GUG/W/23/13361**

Time : Three Hours * 6 7 5 7 * Max. Marks : 60

Notes : 1. Solve all the five questions.
2. Each question carry equal marks.

UNIT – I

1. a) State & prove the Cauchy – Riemann equations in polar form. **6**

b) If the function $f(z) = u + iv$ be analytic in the domain D then show that the families of curves $u(x, y) = c_1$ & $v(x, y) = c_2$ form an orthogonal system, where c_1 and c_2 are arbitrary constants. **6**

OR

c) Show that an analytic function with constant modulus is constant. **6**

d) Show that $u = 2x - x^3 + 3xy^2$ is harmonic & find its harmonic conjugate. **6**

UNIT – II

2. a) Evaluate $\int_c (z - z^2) dz,$ where c is the upper half of the circle $|z| = 1.$ **6**

b) State & prove the Cauchy's integral theorem. **6**

OR

c) Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $|z| = \frac{3}{2}$ **6**

d) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2 + 4)}$ at all its poles in the finite plane. **6**

UNIT – III

3. a) If $\overline{r} = a\cos t\,\overline{i} + a\sin t\,\overline{j} + at\tan\alpha\,\overline{k}$ then find $|\dot{\overline{r}} \times \ddot{\overline{r}}|$ & $[\dot{\overline{r}}, \ddot{\overline{r}}, \dddot{\overline{r}}]$ **6**

b) If $\overline{f} = x^2 z\,\overline{i} - 2y^3 z^2\,\overline{j} + xy^2 z\,\overline{k}$ find div $\overline{f}$ & curl $\overline{f}$ at (1, -1, 1) **6**

OR

c) The acceleration of a particle at any time is $e^t\bar{i} + e^{2t}\bar{j} + \bar{k}$, find the velocity $\bar{v}$ & $\bar{r}$ if $\bar{v}$ & $\bar{r}$ are zero at t = 0. **6**

d) Compute the line integral $\int_c y^2 dx - x^2 dy$ about the triangle whose vertices are (1, 0), (0, 1) & (-1, 0). **6**

UNIT – IV

4. a) Evaluate $\oint_c (y - \cos x)dx + \sin x dy$ using Green's theorem where c is the triangle with vertices at (0, 0), $\left(\frac{\pi}{2}, 0\right) \& \left(\frac{\pi}{2}, 2\right)$ **6**

b) Evaluate $\iint_s \bar{f} \cdot n\, ds$ where $\bar{f} = x^2 i + y^2 j + z^2 k$ & s is the surface of the solid cut off by the plane x + y + z = a from the first octant. **6**

OR

c) Evaluate $\oint_c (ydx + zdy + xdz) \cdot$ by Stoke's theorem, where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2 \; \& \; x + z = a$. **6**

d) State & prove the Gauss divergence theorem. **6**

5. Solve **any six**.

a) Define the bilinear transformation. **2**

b) Find the fixed points of the transformation **2**
$$\omega = \frac{(2+i)z - 2}{i + z}$$

c) Define the Residue. **2**

d) State the Cauchy's integral formula. **2**

e) Define a solenoidal & irrotational vectors. **2**

f) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find curl.grade ϕ. **2**

g) Show that $\iint_s r \cdot n\, ds = 3V$ where V is the volume enclosed by s. **2**

h) State the Green's theorem. **2**

****