B.Sc.-II (CBCS Pattern) Semester - IV

USMT-08 - Mathematics-II Paper- VIII (Elementary Number Theory)

P. Pages : 2 **GUG/S/23/12015**

Time : Three Hours *2059* Max. Marks : 60

Notes : 1. Solve **all five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Let a and b be integers such that b>0. Then prove that there are unique integers q and r such that $a = bq + r$ with $0 \le r < b$. **6**

b) Prove that the product of any m consecutive integers is divisible by m!. **6**

OR

c) Using the Euclidean algorithm, find the gcd d of the numbers 1109 and 4999 and then find integers x and y to satisfy d=1109x+4999y. **6**

d) If c is any common multiple of a and b. Then prove that [a, b]|c **6**

UNIT - II

2. a) Prove that every positive integer greater than one has at least one prime divisor. **6**

b) Show that if m is a composite integer, then prove that $\underbrace{\overleftrightarrow{11\cdots11}}_{\text{m term}}$ is a composite integer. **6**

OR

c) Prove that any two distinct Fermat number are relatively prime i.e. $(f_m, F_n) = 1$. **6**

d) Prove that Diophantine equation $ax + by = c$ has a solution iff d/c, where d = (a, b). **6**

UNIT - III

3. a) Show that if $a \equiv 1 \left(\mod p^n\right)$, then $a^p \equiv 1 \left(\mod p^{n+1}\right)$, where n is a positive integer and p is a prime number. **6**

b) If $r_1, r_2, ..., r_m$ is a complete system of residues modulo m and (a, m) = 1, a is a positive integer, then prove that
$ar_1 + b, ar_2 + b...., ar_m + b$
is also complete system of residues modulo m. **6**

OR

c) Show that the system of congruences $x \equiv a \pmod{m}, x \equiv b \pmod{n}$ has a solution iff $(m,n) | (a-b)$. **6**

d) Solve the congruence $140x \equiv 133 \pmod{301}$. **6**

UNIT - IV

4. a) Let $n = p_1^{a_1} p_2^{a_2} .....p_m^{a_m}$ be the prime-power factorization of the positive integer n. Then prove that **6**

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_m}\right).$$

b) Solve the linear congruence. $3x \equiv 5$ (mod 16) by suing Euler's theorem. **6**

OR

c) Let F and f be two arithmetic functions such that **6**

$$F(n) = \sum_{d/n} f(d)$$

Then prove that $f(n) = \sum_{d/n} \mu(d) F(n/d) = \sum_{d/n} \mu(n/d) F(d)$

d) Show that the integer solution of $x^2 + 2y^2 = z^2$ with $(x, y, z) = 1$ can be expressed as $x = \pm\left(2a^2 - b^2\right)$, $y = 2ab$, $z = 2a^2 + b^2$ **6**

5. Solve **any six.**

a) Prove that $n^3 - n$ is divisible by 6. **2**

b) Prove that there are infinitely many pairs of integers x and y satisfying x+y = 100 and (x, y) = 5. **2**

c) Define a Fermat numbers. **2**

d) Prove that $\left(a^2, b^2\right) = c^2$ if $(a, b) = c$. **2**

e) Define a linear congruence. **2**

f) State the Chinese remainder theorem. **2**

g) If p is prime and $p \nmid a$, then show that $a^{p-1} \equiv 1 \pmod{p}$. **2**

h) Define the Mobius μ function. **2**

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