B.Sc.- II CBCS Pattern Semester-IV

USMT-08 - Mathematics-II Paper-VIII : Elementary Number Theory

P. Pages : 2 **GUG/W/23/12015**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that $(n-1)^2 \mid \left(n^k - 1\right) \Leftrightarrow (n-1) \mid k$, for any positive integer k and an integer $n \geq 2$. **6**

b) If x and y are odd. Prove that $x^2 + y^2$ is not a perfect square. **6**

OR

c) Find the gcd of 275 and 200 express it in the form 275x+200y. **6**

d) Prove that **6**
(a,b) [a,b] = ab, for positive integer a and b.

UNIT – II

2. a) Let a and b be relatively prime integers. If d is a positive divisor of ab. Show that there is a unique pair of positive divisor d_1 of a and d_2 of b such that $d = d_1 d_2$. **6**

b) If (a,b) = 1, then show that $\left(a^2, b^2\right) = 1$ and Prove that $\left(a^2, b^2\right) = c^2$, if (a,b) = c, c > 0. **6**

OR

c) Prove that $F_0 F_1 \cdots F_{n-1} = F_n - 2$, for all positive integer n. **6**

d) Find the solution of linear Diophantine equation 10x+6y=110. **6**

UNIT – III

3. a) Let f denote a polynomial with integral coefficients. If $a \equiv b$ (modm). Prove that $f(a) \equiv f(b)$ (modm). **6**

b) Prove that congruence is an equivalence relation. **6**

OR

c) Show that the system of congruences $x \equiv a$ (modm), $x \equiv b$ (modn) has a solution if and only if $(m,n) \mid (a-b)$. **6**

d) Solve the system of three congruences $x \equiv 2 (\mathrm{mod}\, 3)$, $x \equiv 3 (\mathrm{mod}\, 5)$ and $x \equiv 2 (\mathrm{mod}\, 7)$. **6**

UNIT – IV

4. a) Solve linear congruence $5x \equiv 3 \pmod{14}$ by using Euler's theorem. **6**

b) Prove that the Mobius $\mu -$ function is multiplicative. **6**

OR

c) If x, y, z is Pythagorean triple and (x,y) = d.
Prove that (y,z) = (z,x) = d **6**

d) Let $n = p_1^{a_1} p_2^{a_2} \ldots p_r^{a_r}$ be the prime factorization of the integer n > 1. If f is multiplicative function. Prove that **6**

$$\sum_{d|n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) \ldots (1 - f(p_r)).$$

5. Solve **any six.**

a) Show that if a is an integer, then 3 divides $a^3 - a$. **2**

b) State the Euclidean algorithm. **2**

c) Prove that if $2^m - 1$ is prime, then m is also prime. **2**

d) Define linear Diophantine equation. **2**

e) If $a \equiv b \pmod{m}$ and $d \mid m, d > 0$, then prove that $a \equiv b \pmod{d}$ **2**

f) Find the remainder obtained upon dividing the sum.
1!+2!+3!+4!+5!+ --- +1000!+1001!by 12. **2**

g) Find the units digit of 3^{1000} by Euler's theorem. **2**

h) Define Pythagorean Triple. **2**

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