B.Sc.-II (CBCS Pattern) Semester - IV

USMT-07 - Mathematics-I Paper-VII (Algebra)

P. Pages : 2 | **GUG/S/23/12014**

Time : Three Hours | Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let G be set of all 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d are real number, such that $ad-bc\neq 0$. Show that G is an infinite non abelian group with respect to operation multiplication of matrices. **6**

b) If G is a group, then prove that $(ab)^{-1}=b^{-1}a^{-1}\ \forall a,b\in G$ **6**

OR

c) If H and K are subgroups of G, show that $H\cap K$ is a subgroup of G. **6**

d) For $S=\{1,2,3,4,5,6,7,8,9\}$ and $a=(1\ 3\ 5)(1\ 2),\ b=(1\ 5\ 7\ 9)$ then compute $a^{-1}ba$. **6**

UNIT – II

2. a) Prove that N is a normal subgroup of G if and only if $g^{N}g^{-1}=N\ \forall\ g\in G$. **6**

b) Let $a\in G$ be arbitrary and H be a subgroup of G. Then prove that $Ha=H\Leftrightarrow a\in H$. **6**

OR

c) Let H be a subgroup of G. then prove that there is a one to one correspondence between any two right losets of H in G. **6**

d) If G is a finite group and H is a subgroup of G, then prove that $O(H)$ is a divisor of $O(G)$. **6**

UNIT – III

3. a) G is a group of non zero real numbers under multiplication, $\phi:G\to G$. s.t. $\phi(x)=x^{2}$. Show that ϕ is a homomorphism and determine its Kernel. **6**

b) Let N be a normal subgroup of G. Define mapping $\phi:G\to G/N$ such that $\phi(x)=Nx\ \forall\ x\in G$ then prove that ϕ is a homomorphism of G onto G/N. **6**

OR

c) Let ϕ be a homomorphism of G onto G^1 with Kernel K. Then prove that $G/K \approx G^1$. **6**

d) If ϕ is a homomorphism of G into G^1 with Kernel K, then prove that K is a normal subgroup of G. **6**

UNIT – IV

4. a) Prove that a Ring R is commutative if and only if $(a+b)^2 = a^2 + 2ab + b^2, \ \forall a, b \in R$. **6**

b) If R is a ring with zero element 0, then prove that for all $a, b \in R$ **6**

i) $a\,0 = 0\,a = 0$

ii) $a(-b) = (-a)\,b = -(ab)$

OR

c) Prove that A non empty subset S of a ring R is a subring of R. $\Leftrightarrow x - y, x\,y \in S \ \forall x, y \in S$. **6**

d) Show that the ring $R = \left\{a + b\sqrt{2} \ / \ a, b \in Z\right\}$ is an integral domain under addition and multiplication. **6**

5. Attempt **any six.**

a) Prove that the identity of a group G is unique. **2**

b) If $G = \{1, -1, i, -i\}$ and operation is usual multiplication. Find the order of $(-i)$. **2**

c) Define right coset of H in G. **2**

d) Define Quotient Group. **2**

e) Define Kernel of Homomorphism. **2**

f) If ϕ is an homomorphism of a group G into group G^1 then prove that $\phi(e) = e^1$. **2**

g) Define associative ring. **2**

h) Define zero divisors in ring. **2**

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