B.Sc.-II CBCS Pattern Semester-IV

USMT-07 - Mathematics-I Paper-VII : Algebra

P. Pages : 2 **GUG/W/23/12014**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i for all $a, b \in G$. Then show that G is abelian. **6**

b) Prove that, a nonempty subset H of the group G is a subgroup of G if and only if **6**

i) $a, b \in H \Rightarrow ab \in H$

ii) $a \in H \Rightarrow a^{-1} \in H$

OR

c) If H and K are subgroups of G. Then show that $H \cap K$ is a subgroup of G. **6**

d) For $S = \{1, 2, 3, ----9\}$ and $a, b \in A(S)$ **6**

Find $a^{-1}ba$ where $a = (5\ 7\ 9)$

$b = (1\ 2\ 3)$

UNIT – II

2. a) Prove that, any two right cosets of a subgroups of group G are either disjoint or identical. **6**

b) If G is a finite group and H is a subgroup of G then prove that 0(H) is a divisor of 0(G). **6**

OR

c) Prove that, a subgroup N of group G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G. **6**

d) Let H be a subgroup of group G. Let for $g \in G$, $gHg^{-1} = \{ghg^{-1} / h \in H\}$ **6**

Prove that gHg^{-1} is a subgroup of G.

UNIT – III

3. a) Let G be any group, g a fixed element in G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that ϕ is an isomorphism of G into G. **6**

b) If ϕ is a homeomorphism of G into G^1 with Kernel K then prove that K is normal subgroup of G. **6**

OR

c) Prove that, any infinite cycle group is isomorphic to the additive group of integers. **6**

d) If M, N are normal subgroups of group G **6**

Prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$

UNIT – IV

4. a) If R is a ring with zero element 0, then for all $a, b \in R$, prove that. **6**

i) $a_0 = 0_a = 0$

ii) $a(-b) = (-a)b = -(ab)$

iii) $(-a)(-b) = ab$

b) If in a ring R, $x^3 = x, \forall x \in R$ then show that R is commutative ring. **6**

OR

c) Prove that, the intersection of two subrings is a subring. **6**

d) If R is a ring in which $x^2 = x, \forall x \in R$ then prove that R is a commutative ring of characteristic 2. **6**

5. Solve **any six**.

a) If G is a group then for every $a \in G$ prove that $\left(a^{-1}\right)^{-1} = a$. **2**

b) If a is a generator of a cyclic group G then prove that a^{-1} is also a generator of G. **2**

c) Let $G = \{1, -1, i, -i\}$ and $N = \{1, -1\}$ show that N is a normal subgroup of the multiplicative group G. **2**

d) If G is a finite group and N is a normal subgroup of G then prove that. **2**

$0(G/N) = o(G)/0(N)$

e) Let G be a group of integers under usual addition and $G^1 = G$ **2**

Define $Q : G \to G^1$ by $\phi(a) = na \forall\ a \in G, n \in 2$ then show that ϕ is homomorphism.

f) Define homomorphism and kernel of homomorphism. **2**

g) Let R be a ring, prove that if $a, b \in R$ then $(a+b)^2 = a^2 + ab + ba + b^2$ **2**

h) Define Integral domain. **2**

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