B.Sc.-I (CBCS Pattern) Semester - II

USMT-03 - Mathematics Paper-I

(Ordinary Differential Equations and Difference Equations)

P. Pages: 2

Time: Three Hours

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GUG/S/23/11586

Max. Marks: 60

Notes: 1. Solve all **five** questions.

2. All questions carry equal marks.

UNIT - I

1. a) Show that the DE $(e^y + 1)\cos x \, dx + e^y \sin x \, dy = 0$ is exact and solve it.

b) Solve $y' + y = \frac{1}{1 + e^{2x}}$

OR

Solve DE $xy - \frac{dy}{dx} = y^3 e^{-x^2}$

d) Find the orthogonal trajectories of the family of semicubical parabolas $ay^2 = x^3$.

UNIT - II

2. a) Solve $\frac{1}{D^2 + D} \left(\frac{1}{1 + e^x} \right)$

Solve the DE $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

OR

Find the particular integral of $(D^2 + 4D + 3)y = e^{-3x}$

Solve $\frac{dx}{dt} + 4x + 3y = t$ and $\frac{dy}{dt} + 2x + 5y = e^t$

UNIT - III

3. a) Prove that the Wronskian $W(y_1, y_2, x)$ of any two solutions y_1 and y_2 of the DE x $-\int p(t)dt$ $y'' + py' + qy' = 0, \ p, \ q \in c^o \ \text{satisfies the identify} \ W(y_1, y_2, x) = W(y_1, y_2, a)e^{-a}$

b) Solve $(x^2 D^2 - 3xD + 4)y = 2x^2$

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OR

Solve DE
$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

Solve $y'' + y = \sec^2 x$ by method of variation of parameters.

Solve $y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$.

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UNIT - IV

4. a) From the equation $y_n = A.3^n + B.5^n$, derive a difference equation by eliminating arbitrary constants A and B.

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OR

Solve
$$y_{k+1} + \frac{1}{4}y_k = (1/4)^k$$
, $k \ge 0$, $y(0) = 1$.

d) Solve $y_{n+2} - 2\cos\alpha y_{n+1} + y_n = \cos n\alpha$.

5. Attempt any six.

b)

a) Find integrating factor of
$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

b) Solve $P^2 - 4P + 3 = 0$.

c) Solve
$$(D^3 - 3D^2 + 3D - 1)y = 0$$

d) Find P.I of
$$\frac{1}{D+1}e^{2x}$$

e) At y_1 and y_2 be any two solutions of the DE y'' + py' + qy = 0 if $W(y_1, y_2, x) = 0$ then prove that y_1 and y_2 are linearly dependent.

f) Define Wronskian.

g) Write difference equation
$$\Delta^2 y_n - 3\Delta y_n + 3y_n = 0$$
 in E – form.

h) Define order of a difference equation.
