PSCPHYT01 - (Core-I) Paper-I : Mathematical Physics

_____

Notes : 1. All questions are compulsory.

Either:

1. a) Prove that $\left(y^2 - z^2 + 3yz - 2x\right) i + \left(3xz + 2xy\right) j + \left(3xy - 2xz + 2z\right)k$ is both solenoidal and irrotational. **8**

   b) Find the Fourier series for. **8**
   $$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

   OR

   e) Find sine and cosine transform of. **8**
   $$f(x) = \begin{cases} 1 + x/a & -a < x < 0 \\ 1 - x/a & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

   f) The fluid motion is given by $\bar{v}\left(y \sin z - \sin x\right)i + \left(x \sin z + dyz\right)j + \left(xy \cos z + y^2\right)k$ is the motion is irrotational. If so find velocity potential. **8**

   Either:

2. a) If there be an entity represented by multi suffix set a_{ij} relatively to any given system of rectangular axes and if $a_{ij} b_i$ is a vector, where b_i is any arbitrary vector whatsoever then a_{ij} is a tensor of order two. **6**

   b) i) $\operatorname{grad}\left(\vec{f}.\vec{g}\right) = \vec{f} \times \operatorname{curl} \vec{g} + \vec{g} \times \operatorname{curl} \vec{f} + \vec{f}.\Delta \vec{g} + \vec{g}.\Delta \vec{f}$ **10**

      ii) $\operatorname{curl}\left(\vec{f} \times \vec{g}\right) = \vec{f} \operatorname{div} \vec{g} - \vec{g}.\operatorname{div} \vec{f} + \vec{g}.\Delta \vec{f} - \vec{f}.\Delta \vec{g}$

   OR

   e) If $a_{ijkl......}$ is a tensor of order m then set obtained an identifying any two suffixes is a tensor of order (m-2). **6**

   f) Show that if $a_{ijkl......}$ is symmetric (skew symmetric) in any two suffixes, then so is also $\bar{a}_{pqrs.....}$ in the same suffix. **6**

g) What is mixed tensor of second rank. Prove that δ_q^p is a mixed tensor of the second rank. **4**

Either:

3. a) Find eigen value of A^3 if. **6**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Satisfy the Cayley-Hamilton theorem and find A^{-1} of the matrix. **10**

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$$

OR

e) Find the eigen value & eigen vector of matrix. **8**

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

f) Diagonalise $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **8**

Either:

4. a) Solve the differential equations. **8**

i) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$.

ii) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 6y = e^x \cosh 2x$.

b) $3x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$ **8**

Find the solution in generalized series form about $x = 0$ of differential equation.

OR

e)   Prove that.

    i) $xJ_n' = nJ_n - xJ_{n+1}$ **4**

    ii) $xJ_n' = -nJ_n + xJ_{n-1}$ **4**

f)   Find the power series solution of.   **8**

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

5.   Answer the following.

a)   Find the Fourier expansion of time period of 2π, where $f(x) = x^2$, $-\pi < x < \pi$.   **4**

b)   Prove that $\in_{i\ell m}\in_{jem} = 2\delta_{ij}$ **4**

c)   Find A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 1 & 2 \end{bmatrix}$ **4**

d)   Solve differential equation $\sin x\dfrac{dy}{dx} + 2y = \tan^3(x/2)$.   **4**

****