## M.Sc.- II (Mathematics) (NEW CBCS Pattern) Sem-IV

## PSCMTH20: Foundation Course : Operations Research-II

P. Pages : 4

GUG/W/22/13775
Time : Three Hours

Notes: 1. Solve all five questions.
2. All questions carry equal marks.

## UNIT - I

1. a) Find the optimum integer solution to the following L.P.P.

Maximize $Z=x_{1}+4 x_{2}$
subject to the constraints,
$2 \mathrm{x}_{1}+4 \mathrm{x}_{2} \leq 7$,
$5 x_{1}+3 x_{2} \leq 15$
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$ and are integers
b) Using the cutting-plane algorithm solve the integer programming problem.

Maximize $Z=3 x_{1}+x_{2}+3 x_{3}$
subject to the constraints,
$-\mathrm{x}_{1}+2 \mathrm{x}_{2}+\mathrm{x}_{3} \leq 4$
$4 x_{2}-3 x_{3} \leq 2$
$\mathrm{x}_{1}-3 \mathrm{x}_{2}+2 \mathrm{x}_{3} \leq 3$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}$ are non - negative integer

## OR

c) Find the optimum integer solution to the following all integer programming problem
subject to the constraints,
$-\mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 6$
$7 \mathrm{x}_{1}+\mathrm{x}_{2} \leq 35$
$x_{1}, x_{2} \geq 0$ and $x_{1}$ in an integer
d) Solve the IPP

Minimize $Z=9 x_{1}+10 x_{2}$
subject to
$\mathrm{x}_{1} \leq 9$
$\mathrm{x}_{2} \leq 8$
$4 x_{1}+3 x_{2} \geq 40$
$x_{1}, x_{2} \geq 0$ and are integer

## UNIT - II

2. a) Solve the following linear goal programming problem graphically.

Find $x_{1} \& x_{2}$ so as to :
Minimize $\mathrm{Z}=\mathrm{G}_{1}\left(\mathrm{~d}_{3}^{+}+\mathrm{d}_{4}^{+}\right)+\mathrm{G}_{2} \mathrm{~d}_{1}^{+}+\mathrm{G}_{3} \mathrm{~d}_{2}^{-}+\mathrm{G}_{4}\left(\mathrm{~d}_{3}^{-}+\frac{3}{2} \mathrm{~d}_{4}^{-}\right)$
and satisfy the goals :
$\mathrm{G}_{1}: \mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{d}_{1}^{-}+\mathrm{d}_{1}^{+}=40$
$\mathrm{G}_{2}: \mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{d}_{2}^{-}-\mathrm{d}_{2}^{+}=100$
$\mathrm{G}_{3}: \mathrm{x}_{1}+\mathrm{d}_{3}^{-}-\mathrm{d}_{3}^{+}=30$
$\mathrm{G}_{4}: \mathrm{x}_{2}+\mathrm{d}_{4}^{-}-\mathrm{d}_{4}^{+}=15$
$\mathrm{x}_{\mathrm{i}}, \mathrm{d}_{\mathrm{i}}^{-}, \mathrm{d}_{\mathrm{i}}^{+} \geq 0$ for all $\mathrm{i}=1,2,3,4$.
The goals have been listed in order of priority.
b) Use revised simplex method to solve the following L.P.P. :

Maximize $\mathrm{Z}=3 \mathrm{x}_{1}+5 \mathrm{x}_{2}$
subject to constraints
$\mathrm{x}_{1} \leq 4, \mathrm{x}_{2} \leq 6,3 \mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 18 \&$
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0, \mathrm{x}_{3} \geq 0$

## OR

c) Using the bounded variable technique, solve the following L.P.P. :

Maximize $Z=x_{2}+3 x_{3}$
subject to constraints
$\mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3} \leq 10, \mathrm{x}_{1}-2 \mathrm{x}_{3} \geq 0$
$2 x_{2}-x_{3} \leq 10 \quad, 0 \leq x_{1} \leq 8$
$0 \leq x_{2} \leq 4 \quad, x_{3} \geq 0$
d) For the following L.P.P. :

Maximize $\mathrm{Z}=(3-6 \lambda) \mathrm{x}_{1}+(2-2 \lambda) \mathrm{x}_{2}+(5+5 \lambda) \mathrm{x}_{3}$
subject to constraints
$x_{1}+2 x_{2}+x_{3} \leq 430$,
$3 x_{1}+2 x_{3} \leq 460$,
$\mathrm{x}_{1}+4 \mathrm{x}_{2} \leq 420$,
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0$
Find the range of $\lambda$ over which the solution remains basic feasible \& optimal.
3. a) Use graphical method to find the minimum elapsed total time sequence of 2 jobs and 5 machines, when we are given the following information :

## Machines

Job $1\left\{\begin{array}{cccccc}\text { sequence: } & \text { A } & \text { B } & \text { C } & \text { D } & \text { E } \\ \text { Time (in hours): } & 2 & 3 & 4 & 6 & 2\end{array}\right.$
Job $2\left\{\begin{array}{cccccc}\text { sequence: } & \text { C } & \text { A } & \text { D } & \text { E } & \text { B } \\ \text { Time(in hours): } & 4 & 5 & 3 & 2 & 6\end{array}\right.$
b) In a factory there are 6 Jobs to perform each of which should go through two machines A and B in the order A, B. The processing timings (in hrs) for the jobs are given below. Determine the sequence for performing the jobs that would minimize the total elapsed time T and what is the value of T ?

| Job | $\mathrm{J}_{1}$ | $\mathrm{~J}_{2}$ | $\mathrm{~J}_{3}$ | $\mathrm{~J}_{4}$ | $\mathrm{~J}_{5}$ | $\mathrm{~J}_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine A | 1 | 3 | 8 | 5 | 6 | 3 |
| Machine B | 5 | 6 | 3 | 2 | 2 | 10 |

## OR

c) A TV repairman finds that the time on his jobs has an expotential distribution with mean, 30 minutes. If he repair sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hours day. What is repairman's expected idle time each day? How many job are ahead of average set just brought in?
d) Determine the optional sequence of jobs that minimizes the total elapsed time based on following information processing time on machines is given in hours \& passing is not allowed.

| Job | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine $\mathrm{M}_{1}$ | 3 | 8 | 7 | 4 | 9 | 8 | 7 |
| Machine $\mathrm{M}_{2}$ | 4 | 3 | 2 | 5 | 1 | 4 | 3 |
| Machine $\mathrm{M}_{3}$ | 6 | 7 | 5 | 11 | 5 | 6 | 12 |

## UNIT - IV

4. a) Obtain the necessary and sufficient conditions for the optimum solution of the following NLPP.
Minimize $\mathrm{Z}=\mathrm{f}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right)=3 \mathrm{e}^{2 \mathrm{x}_{1}+1}+2 \mathrm{e}^{\mathrm{x}_{2}+5}$
subject to constraints
$\mathrm{x}_{1}+\mathrm{x}_{2}=7$ and $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
b) Obtain the set of necessary \& sufficient condition for the non-linear programming problem
Minimize $Z=2 x_{1}^{2}-24 x_{1}+2 x_{2}^{2}-8 x_{2}+2 x_{3}^{2}-12 x_{3}+200$
subject to constraints
$x_{1}+x_{2}+x_{3}=11$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0$

## OR

c) Use Kuhn-Tucker condition to solve the NLPP.

Maximize $Z=7 x_{1}^{2}+6 x_{1}+5 x_{2}^{2}$
subject to constraints
$\mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 10$
$\mathrm{x}_{1}-3 \mathrm{x}_{2} \leq 9$
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
d) Solve the following non-linear programming problems, using the method of Lagrangian multipliers,
Minimize $Z=6 x_{1}^{2}+5 x_{2}^{2}$
subject to constraints
$x_{1}+5 x_{2}=3$
$\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$
5. a) What is an integer programming problem. 5
b) Write the major steps in the formulation of linear goal programming problem.
c) Write the basic terms used in sequencing.
d) Define general non-linear programming problem.

