M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-IV

PSCMTH19 (C) : Core Elective Course : Combinatorics

P. Pages : 2 **GUG/W/22/13772**

Time : Three Hours * 3 4 7 6 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) A committee of k people is to be chosen from a set of seven women & four men. **10**
How many ways are there to form the committee if.
a) The committee consists of three women & two men?
b) The committee can be any positive size but must have equal numbers of women and men?
c) The committee has four people and one of them must be Mr. Baggins?

b) What is the probability that a 4-digit campus telephone number has one or more repeated digits? **10**

OR

c) How many ways are there to form a three letter sequence using the letters a, b, c, d, e, f **10**
a) with repetition of letters allowed?
b) without repetition of any letter?
c) without repetition and containing the letter e?

d) How many ways are there to arrange the seven letters in the word SYSTEMS? In how many of these arrangements do the three Ss appear consecutively? **10**

UNIT – II

2. a) Use generating functions to find the number of ways to collect \$15 from 20 distinct people if each of the first 19 people can give a dollar (or nothing) & the twentieth person can give either \$1 or \$5 (or nothing) **10**

b) Find the coefficient of x^{16} in $\left(x^2+x^3+x^4+----\right)^5$. What is the coefficient of x^r ? **10**

OR

c) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? **10**

d) Find the coefficient of x^{24} in $\left(x+x^2+x^3+x^4+x^5\right)^8$. **10**

UNIT – III

3. a) Solve the recurrence relation **10**

$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}, a_0 = a_1 = 1,\ a_2 = 2$

b) Solve the recurrence relation **10**

$a_n = 3a_{n-1} + 4a_{n-2},\ a_0 = a_1 = 1$

OR

c) Solve the recurrence relation **10**

$a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0 = a_1 = 1.$

d) Every year Dr. Finch's rabbit population doubles. He started with six rabbits. How many rabbits does he have after eight years? After n years? **10**

UNIT – IV

4. a) If a school has 100 students with 40 taking French, 40 taking Latin, and 40 taking German, 20 students are taking any given pair of languages and 10 students are taking all three languages, then how many students are taking no language? **10**

b) How many positive integers ≤ 70 are relatively prime to 70? **10**

OR

c) State & prove Inclusion-Exclusion formula. **10**

d) How many ways are there to distribute r distinct objects into five distinct boxes with at least one empty box? **10**

5. a) How many arrangements of letters in REPETITION are there? **5**

b) Find a generating function for a_r, the number of ways to distribute r identical objects into Five distinct boxes with an even number of objects not exceeding 10 in the first two boxes & between three and five in the other boxes. **5**

c) Find a recurrence relation for the number of ways to arrange n distinct objects in a row. Find the number of arrangements of eight objects. **5**

d) If a school has 150 students with 60 students taking French, 30 students taking Latin, 30 students taking both languages, how many students take no language? **5**

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