M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-IV

PSCMTH18: Paper-III : Integral Equations

P. Pages : 2 **GUG/W/22/13769**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Show that $u(x) = \cos 2x$ is solution of the integral equation **10**

$$u(x) = \cos x + 3\int_0^x k(x,t), u(t)\,dt$$

Where $k(x,t) = \begin{cases} \sin x \cos t, & 0 \le x \le t \\ \cos x \sin t, & t \le x \le \pi \end{cases}$

b) Obtain an integral equation for the DE $y'' - 2xy = 0$ with initial conditions $y(0) = \frac{1}{2},\ y'(0) = 1, y''(0) = 1$. **10**

OR

c) Convert $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with $y(0) = 1, y'(0) = -2$ into an Volterra integral equation of 2nd kind. **10**

d) Obtain the integral equation from the DE $\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)\,y = x\,e^x + 1$ with $y(0) = y'(0) = 1,\ y''(0) = 0$. **10**

UNIT – II

2. a) Find the eigen value & eigen function of the homogeneous integral equation **10**

$$u(x) = \lambda\int_{-1}^{1}\left(5xt^3 + 4x^2t + 3tx\right)u(t)\,dt$$

b) Solve : $u(x) = x + \lambda\int_0^1 (1 + x + t)\,u(t)\,dt$ **10**

OR

c) Solve the integral equation **10**

$$u(x) = f(x) + \lambda\int_0^1 (1 - 3xt)\,u(t)\,dt$$

d) Show that the equation **10**

$u(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\,u(t)\,dt$ possesses no solution for f (x) = x but it possesses infinitely many solutions when f (x) = 1.

UNIT – III

3. a) State & prove the Bessel's inequality. **10**

b) Show that the set of eigen values of the 2nd iterated Kernel coincide with the set of squares of the eigen values of the given Kernel. **10**

OR

c) Solve : $u(x) = \cos 3x + \lambda \int_0^{\pi} \cos(x+t)\, u(t)\, dt$ **10**

d) Solve : $u(x) = e^x + \lambda \int_0^1 \left(5x^2 - 3\right) t^2\, u(t)\, dt$ **10**

UNIT – IV

4. a) Solve the Fredholm equation of second kind $u(x) = 2x + \lambda \int_0^1 (x+t)\, u(t)\, dt$ by method of successive approximation by taking $u_0(x) = 1$. **10**

b) Solve : $u(x) = f(x) + \frac{1}{2} \int_0^1 e^{x-t}\, u(t)\, dt$ **10**

OR

c) Solve : $u(x) = \cos x - x - 2 + \int_0^x (t-x)\, u(t)\, dt$ **10**

d) Solve the Volterra integral equation of first kind $f(x) = \int_0^x e^{x-t}\, u(t)\, dt, f(0) = 0$ **10**

5. a) Show that $u(x) = 1 - x$ is a solution of the integral equation $\int_0^x e^{x-t}\, u(t)\, dt = x$. **5**

b) Find C_1 for the homogeneous integral equation of 2nd kind $u(x) = \lambda \int_0^{2\pi} \sin(x+t)\, u(t)\, dt$, in terms of C_2. **5**

c) State the Hilbert – Schmidt theorem. **5**

d) Solve the equation $u(x) = 1 + x - \int_0^x u(t)\, dt, u_0(x) = 1$ by successive approximation method. **5**

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