M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-IV

PSCMTH17 : Partial Differential Equations

P. Pages : 3 **GUG/W/22/13768**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that : The singular integral of $f(x, y, z, p, q) = 0$ satisfies the following equations. **10**

$f(x, y, z, p, q) = 0$

$f_p(x, y, z, p, q) = 0$

$f_q(x, y, z, p, q) = 0$

b) Find the general integral of **10**
$yzp + xzq = x + y$

OR

c) Prove that: If $\vec{x} \cdot \text{curl}\,\vec{x} = 0$ where $\vec{x} = (P, Q, R)$ and μ is an arbitrary differentiable function of x, y and z, then **10**
$\mu\vec{x}.\text{curl}\left(\mu\vec{X}\right) = 0$.

d) Solve the equation **10**
$z^2 + zu_z - u_x^2 - u_y^2 = 0$
by Jacobi's method

UNIT – II

2. a) Find a complete integral of the equation **10**
$\left(p^2 + q^2\right)x = pz$
and the integral surface containing the curve $C: x_0 = 0,\ y_0 = s^2,\ z_0 = 2s$.

b) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x),\ x \geq 0$. **10**

OR

c) Consider the P.D.E. **10**
$f(x, y, z, p, q) = 0$
where f has continuous second order derivatives with respect to its variables x, y, z, p and q, and at every point either $f_p \neq 0$ or $f_q \neq 0$. Suppose that the initial values $z = z_0(s)$ are

specified along the initial curve $\Gamma_0 : x = x_0(s),\ y = y_0(s),\ a \le s \le b$, where $x_0(s),\ y_0(s)$ and $z_0(s)$ have continuous second order derivatives. Suppose $p_0(s)$ and $q_0(s)$ have been determined such that

$$f\ (x_0(s),\ y_0(s),\ z_0(s),\ p_0(s),\ q_0(s) = 0,\ \text{and}\ \frac{dz_0}{ds} = p_0 \frac{dx_0}{ds} + q_0 \frac{dy_0}{ds},$$

where p_0 and q_0 are continuously differentiable functions of S. If, in addition, the five functions $x_0,\ y_0,\ z_0,\ p_0$ and q_0 satisfy

$$f_q \frac{dx_0}{ds} - f_p \frac{dy_0}{ds} \ne 0,$$

then prove that in some neighbourhood of each point of the initial curve there exists one and only one solution $z = z(x, y)$ of the give P.D.E. such that

$$z(x_0(s),\ y_0(s)) = z_0(s)$$

$$z_x(x_0(s),\ y_0(s)) = p_0(s)$$

$$z_y(x_0(s), y_0(s)) = q_0(s)$$

d) Find the solution of the equation **10**

$$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$$

which passes through the x-axis.

UNIT – III

3. a) Derive a linear one-dimensional wave equation governing small transverse vibrations of string. **10**

b) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to a canonical form. **10**

OR

c) Obtain D Alembert's solution of the one dimensional wave equation which describes vibrations of an infinite string. **10**

d) Prove that for the equation **10**

$$Lu = u_{xy} + \frac{1}{4} u = 0$$

the Riemann function is

$$\nu(x, y; \alpha, \beta) = J_0\left(\sqrt{(x - \alpha)(y - \beta)}\right),$$

where J_0 denotes the Bessel's function of the first kind of order zero.

UNIT – IV

4. a) Suppose that u (x, y) is harmonic in a bounded domain D and is continuous in $\overline{D} = D \cup B$. Then u attains its maximum on the boundary B of D. **10**

b) Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula. **10**

OR

c) State and prove Harnack's theorem. **10**

d) Show that the surfaces. **10**

$x^2 + y^2 + z^2 = cx^{2/3}$

can form an equipotential family of surfaces, and find the general form of the potential function.

5. a) Eliminate the parameters a and b from the equation **5**

$z = (x+a)(y+b)$

to find the corresponding P.D.E.

b) Show that there exist a unique solution of $2z_x + yz_y = z$ for the initial data curve **5**

$C : x_0 = s,\ y_0 = s^2,\ z_0 = s,\ 1 \le s \le 2$.

c) Define: **5**

i) Second order semi-linear P.D.E.

ii) Regular solution of second order semi-linear P.D.E.

d) Let D be a bounded domain in R^2, bounded by a smooth closed curve B. Let $\{u_n\}$ be a sequence of functions each of which is continuous on $\overline{D} = D \cup B$ and harmonic in D. If $\{u_n\}$ converges uniformly on B, then prove that $\{u_n\}$ converges uniformly on $\overline{D}$. **5**

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