M.Sc.(Mathematics) (New CBCS Pattern) Semester - III

PSCMTH13 - Paper-III : Mathematical Methods

P. Pages : 2 **GUG/S/23/13757**

Time : Three Hours * 2 4 9 3 * Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carry equal marks.

UNIT - I

1. a) State & prove the modulation theorem for the Fourier transform. **10**

b) Find the Fourier transform of **10**

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases} \text{ \& hence evaluate } \int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}\,dx$$

OR

c) State & prove the convolution theorem for the Fourier transform. **10**

d) Find the Fourier sine transform of $f(x)=\dfrac{1}{x\left(x^2+a^2\right)}$ **10**

UNIT - II

2. a) Solve the wave equation $\dfrac{\partial^2 u}{\partial x^2}=\dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$, $0\le x\le a, t>0$ satisfying the boundary condition $u(0,t)=u(a,t)=0,\ t>0$ & the initial condition $u(x,0)=\dfrac{4b}{a^2}x(a-x),\ \dfrac{\partial u(x,0)}{\partial t}=0,\ 0\le x\le a$ to determine the displacement u(x,t). **10**

b) The transverse displacement of elastic membrane u (x, y, t) satisfies the PDE $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=\dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$, under the boundary conditions u = 0 on the boundary, u = f (x, y), ut = g (x, y) at t = 0. Find the displacement u (x, y, t) after utilizing finite Fourier transform. **10**

OR

c) Let f (x) be continuous & $f'(x)$ be sectionally continuous on the interval $0\le x\le a$ then show that **10**

i) $\bar{f}_c\,[f'(x), x\to n]=(-1)^n f(a)-f(0)+\dfrac{n\pi}{a}\bar{f}_s(n),\ n\in Z^*$

ii) $\bar{f}_s\,[f'(x), x\to n]=-\dfrac{n\pi}{a}\bar{f}_c(n),\ n\in N$

d) The temperature u (x, t) at any point x at any time t in a solid bounded by planes x = 0 & x=4 satisfies the heat condition equation $\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial u}{\partial t}$, when the and faces x = 0 & x =4 are kept at zero temperature. Initially the temperature at x is 2x then find u (x,t) for all x & t. **10**

UNIT - III

3. a) State & prove the first & second shifting theorems of Laplace transform. **10**

b) If Laplace transform of a piecewise continuous function f (t) is $\bar{f}(p)$ then show that **10**

$L\left[t^n f(t)\right] = (-1)^n \frac{d^n \bar{f}(p)}{dp^n}, n$ is the integer.

OR

c) Find $L[E_r f(t); t \to p]$ to show that its value is expressed as $\frac{1}{p} e^{p^2/4} \operatorname{Erfc}\left(\frac{1}{2}p\right)$. **10**

d) Evaluate $L^{-1}\left[\frac{1}{\sqrt{p}\,(p-a)}\right]$ by convolution theorem Hence find $L^{-1}\left[\frac{1}{p\sqrt{p+a}}\right]$ **10**

UNIT - IV

4. a) State & prove the Parseval theorem for the Hankel transform. **10**

b) If $f(x) = \begin{cases} x^n, & x < x < a \\ 0, & x > a \end{cases}$ **10**

Find the Hankel transform of order n of f(x).

OR

c) State & prove the convolution theorem of Mellin transform. **10**

d) If $M\left[f_{(x)}, x \to s\right] = f^*(s)$ then prove that: **10**

i) $M\left[\left(x\frac{d}{dx}\right) f(x); x \to s\right] = -sf^*(s)$.

ii) $M\left[\left(x\frac{d}{dx}\right)^2 f(x); x \to s\right] = s^2 f^*(s)$.

iii) $M\left[\left(x\frac{d}{dx}\right)^n f(x), x \to s\right] = (-1)^n s^n f^*(s), n = 2, 3, ...$

5. a) State & prove the linearity property of Fourier transform. **5**

b) Define finite Fourier cosine & sine transform. **5**

c) State and prove change of scale property of Laplace transform. **5**

d) Define: **5**

i) Hankel transform ii) Mellin transform

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