M.Sc. II (Mathematics) (NEW CBCS Pattern) Sem-III

Core Course - PSCMTH13 - Paper-III : Mathematical Methods

P. Pages : 3 **GUG/W/22/13757**

Time : Three Hours *3461* Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Define the Fourier Transforms and If C_1 and C_2 are constants, then show that **10**

$$F[C_1f_1(x)+C_2f_2(x)]=C_1F[f_1(x)]+C_2F[f_2(x)]$$

b) i) If $F[f(x):x\to\xi]=F(\xi)$, then show that **10**

$$F[f(ax);x\to\xi]=\frac{1}{a}F\left(\frac{\xi}{a}\right)$$

ii) If $F_s[f(x)]=F_s(\xi)$, then prove that

$$F_s[f(ax)]=\frac{1}{a}F_s\left(\frac{\xi}{a}\right)$$

OR

c) If $F(\xi)$ is Fourier Transform of $f(x)$ in R, then show that Fourier Transform of $f(x-a)$ is $e^{i\xi a}\cdot F(\xi)$. **10**

d) Show that $\int_0^\infty \frac{\cos px}{1+p^2}dp=\frac{\pi}{2}e^{-x}, x\ge 0$ **10**

UNIT – II

2. a) Let $f(x)$ be continuous and $f'(x)$ be sectionally continuous on the interval $0\le x\le a$, then prove that **10**

i) $\bar{f}_c[f'(x);x\to n]=(-1)^n f(a)-f(0)+\frac{n\pi}{a}\bar{f}_s(n),\ n\in Z^*$

ii) $\bar{f}_s[f'(x);x\to n]=\frac{-n\pi}{a}\bar{f}_c(n),\ n\in N$

b) Let $f(x)$ and $f''(x)$ be continuous and $f''(x)$ be sectionally continuous in $0\le x\le a$. Then prove that **10**

i) $\bar{f}_c[f''(x);n]=-f'(0)+(-1)^n\cdot f'(a)-\frac{n^2\pi^2}{a^2}\bar{f}_c(n)$

ii) $\bar{f}_s[f''(x);n]=\frac{n\pi}{a}f(0)-(-1)^n\frac{n\pi}{a}f(a)-\frac{n^2\pi^2}{a^2}\bar{f}_s(n)$

OR

c) Solve the wave equation **10**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ 0 \le x \le a,\ t > 0$$

Satisfying the boundary condition $u(0,t) = u(a,t) = 0,\ t > 0$ and the initial condition,

$$u(x,0) = \frac{4b}{a^2}x(a-x), \frac{\partial u(x,0)}{\partial t} = 0,$$

$0 \le x \le a$ to determine the displacement $u(x,t)$.

d) Solve the diffusion equation **10**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t},\ 0 < x < a,\ t > 0$$

for its solution satisfying the boundary condition

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(a,t)}{\partial x} = 0,\ t > 0$$

and the initial condition $u(x,0) = f(x),\ 0 \le x \le a$, using proper finite Fourier Transform.

UNIT – III

3. a) If the Laplace transform of $f(t)$ is $\bar{f}(p)$, then show that Laplace transform of $f(t-a)H(t-a)$ is $e^{-ap} \cdot \bar{f}(p)$. **10**

b) If Laplace transform of a piecewise continuous function $f(t)$ is $\bar{f}(p)$, then show that Laplace transform of $t^n f(t) = (-1)^n \dfrac{d^n \bar{f}(p)}{dp^n}$, when n is positive integer. **10**

OR

c) Define the convolution of two functions and evaluate the Laplace transform of the convolution of two functions. **10**

d) Evaluate **10**

$$L^{-1}\left[\frac{P}{(P^2+4)^3}\right]$$

by using convolution theorem.

UNIT – IV

4. a) If $\bar{f}_n(\xi)$ be Hankel transform of order n of $f(r)$, then prove that Hankel transform of order n of **10**

$$f''(r) + \frac{1}{r}f'(r) - \frac{n^2}{r^2}f(r) \text{ is } -\xi^2 \bar{f}_n(\xi)$$

provided that $r f'(r) \to 0$ and $r f(r) \to 0$ when $r \to 0$ and when $r \to \infty$.

b) Evaluate **10**

a) $H_1\left[\frac{e^{-ay}}{x};\xi\right]$

b) $H_0\left[\frac{1}{x};\xi\right]$

OR

c) State and prove the convolution theorem of Mellin Transform. **10**

d) Find the following Mellin transform of derivative of a function **10**

i) $M\left[f'(x);x\to s\right]=-(s-1)f^{*}(s-1)$

ii) $M\left[f''(x);x\to s\right]=(s-1)(s-2)f^{*}(s-2)$

5. a) Find the Fourier transform **5**

$f(x)=e^{-a|x|},\ a>0$

b) Define the Finite Fourier Cosine and Sine transforms. **5**

c) Find the Laplace transform of sinax. **5**

d) Find the Hankel transform of order zero of **5**

$$f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$$

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