M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-III

PSCMTH12 - Paper-II : Functional Analysis

P. Pages : 2 **GUG/W/22/13756**

Time : Three Hours * 3 4 6 0 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each questions carries equal mark

UNIT – I

1. a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x +M in the quotient space N/M is defined by $\|x+M\| = \inf\{\|x+m\| : m \in M\}$ then prove that N/M is a normed linear space. **10**

b) If M is a closed linear subspace of a normed linear space N and x_o is a vector not in M, then prove that there exists a functional f_o in N^* such that $F_o(M)=0$ and $F_o(x_o) \neq 0$ **10**

OR

c) State and prove Hahn – Banach theorem. **10**

d) Prove that if N is a normed linear space, then the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak topology. **10**

UNIT – II

2. a) If B and B' are Banach spaces and if T is a continuous linear transformation of B onto B', then prove that T is an open mapping. **10**

b) Prove that a non – empty subset X of a normed linear space N is bounded $\Leftrightarrow f(X)$ is a bounded set of numbers for each f in N^*. **10**

OR

c) Prove that, a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm **10**

d) Prove that, if $\{e_i\}$ $i=1,2,----,n$ is an orthonormal set in a Hilbert space H then **10**

$$\sum_{i=1}^{n} |(x, e_i)|^2 \le \|x\|^2, \text{ for every vector x in H}$$

UNIT – III

3. a) Let H be a Hilbert space and let f be a arbitrary functional in H^*. Then prove that there exists a unique vector y in H such that $f(x)=(x,y)$, for every x in H. **10**

b) If T is an operator on H for which $(Tx, x) = 0$, for all x then prove that T = 0. **10**

OR

c) If N_1 and N_2 are normal operator on H with the property that either commutes with the adjoint of the other then prove that $N_1 + N_2$ and $N_1 . N_2$ are normal **10**

d) If T is an operator on Hilbert space H then prove that, T is normal if and only if its real and imaginary parts commute. **10**

UNIT – IV

4. a) Let T be an operator on H then prove that if A is nonsingular then $6\left(ATA^{-1}\right) = 6(T)$ **10**

b) If T is normal then prove that the M_i's are pairwire orthogonal. **10**

OR

c) If T is normal then prove that M_i's span H **10**

d) Show that an operator T on H is normal if and only if its adjoint T^* is polynomial in T. **10**

5. a) Define: **5**

i) Normed linear space

ii) Banach space.

b) Prove that, if $\{e_i\}$ is an orthonormal set in a Hilbert space H and if x is any vector in H then the set $S = \{e_i : (x, e_i) \neq 0\}$ is either empty or countable. **5**

c) If T_1^*, and T_2^* are adjoint operation of T_1 and T_2 then prove that $(T_1 + T_2)^* = T_1^* + T_2^*$ **5**

d) Prove that, if T is normal then each M_i reduces T. **5**

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