M.Sc.(Mathematics) (New CBCS Pattern) Semester - III

PSCMTH11 - Paper-I : Complex Analysis

P. Pages : 2 **GUG/S/23/13755**

Time : Three Hours * 2 4 9 1 * Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. Each question carry equal marks.

UNIT – I

1. a) State & prove the Cauchy-Riemann equations. **10**

b) Define the Harmonic function. Show that if a function $f(z)=u(x,y)+iv(x,y)$ is analytic in a domain D then its component functions u & v are harmonic in D. **10**

OR

c) State & prove the reflection principle. **10**

d) Show that; **10**

i) $\exp\left(\frac{2+\pi i}{4}\right)=\sqrt{\frac{e}{2}}\,(1+i)$ ii) $\log\,(i^3)\neq 3\log i$.

UNIT – II

2. a) Let f be analytic everywhere inside & on a simple closed contour C taken in the positive sense. If z_0 is any interior point to C then show that **10**

$$f(z_0)=\frac{1}{2\pi i}\int_C \frac{f(z)dz}{z-z_0}$$

b) State & prove the maximum modulus principle. **10**

OR

c) State & prove the Taylor's theorem. **10**

d) State & prove the Laurent's theorem of the series representation as **10**

$$f(z)=\sum_{n=0}^{\infty} a_n(z-z_0)^n+\sum_{n=1}^{\infty}\frac{bn}{(z-z_0)^n}\ .$$

UNIT – III

3. a) Let C be a simple closed contour described in the positive sense. If a function f is analytic inside & on C except for a finite number of singular points Z_K inside C then show that **10**

$$\int_C f(z)dz=2\pi i\sum_{K=1}^{n}\operatorname*{Res}_{z=z_K}\ (f(z))$$

b) Evaluate $\int_C \frac{4z-5}{z(z-1)} dz$ by using the Cauchy's residue theorem. **10**

OR

c) Evaluate : $\int_0^\infty \frac{x^2}{x^6+1} dx$. **10**

d) State & prove the Rouche's theorem. **10**

UNIT – IV

4. a) Explain the transformation $w = \frac{1}{z}$ & show that $\lim_{z \to z_0} T(z) = T(z_0)$. **10**

b) Discuss the linear fractional transformations. **10**

OR

c) Find the special case of the linear fractional transformations that maps the points $z_1 = -1,\ z_2 = 0 \ \& \ z_3 = 1$ on the points $w_1 = -i,\ w_2 = 1 \ \& \ w_3 = i$. **10**

d) Show that the transformation $w = \log \frac{z-1}{z+1}$ transforms the plane y > 0 onto the strip $0 < v < \pi$, **10**

5. a) State the sufficient condition for the differentiability. **5**

b) Show that $\int_0^{\frac{\pi}{4}} e^{it} dt = \frac{1}{\sqrt{2}} + i\left(1 - \frac{1}{\sqrt{2}}\right)$. **5**

c) Find the pole & residue of $f(z) = \frac{z^3+2z}{(z-i)^3}$. **5**

d) Show that the transformation $w = iz + i$ maps the half plane x > 0 onto the half plane v > 1. **5**

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