M.Sc. II (Mathematics) (NEW CBCS Pattern) Sem-III

PSCMTH11: Complex Analysis

P. Pages : 2 **GUG/W/22/13755**

Time : Three Hours * 3 4 5 9 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let $f(z) = u(x, y) + iv(x, y)$ $(z = x + iy)$ and $z_0 = x_0 + iy_0$, $w_0 = u_0 + iv_0$. **10**

If $\lim\limits_{(x,y)\to(x_0,y_0)} u(x, y) = u_0$ and $\lim\limits_{(x,y)\to(x_0,y_0)} V(x, y) = V_0$

Then prove that $\lim\limits_{z\to z_0} f(z) = w_0$, and converse also hold.

b) If $f'(z) = 0$ everywhere in a domain D, then f(z) must be constant throughout D. **10**

OR

c) Prove that the zeros of sin z and cos z in the complex plane are the same as the zeros of sin x and cos x on the real line. **10**

d) Suppose that **10**

i) a function f is analytic throughout a domain D.

ii) f(z) = 0 at each point z of a domain or line segment contained in D.

Then prove that f(z) $\equiv 0$ in D.

UNIT – II

2. a) If w(t) is a piecewise continuous complex – valued function defined on an interval $a \le t \le b$, then **10**

$$\left| \int_a^b w(t)\,dt \right| \le \int_a^b | w(t) |\,dt$$

b) If a function f is analytic throughout a simply connected domain D, then $\int_c f(z)dz = 0$ for every closed contour C lying in D. **10**

OR

c) If Z_1 is a point inside the circle of convergence $| Z - Z_0 | = R$ of a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$, then prove that the series must be uniformly convergent in the closed disk $| z - z_0 | \le R_1$, where $R_1 = | z_1 - z_0 |$. **10**

d) Suppose that $z_n = x_n + iy_n \ (n = 1, 2, ...)$ and S = X + iY. Then prove that $\sum_{n=1}^{\infty} Z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$. **10**

UNIT – III

3. a) State and prove Cauchy's Residue theorem. **10**

b) Let f denote a function that is analytic at a point z_0. Then prove that the following two statements are equivalent : **10**

i) f has a zero of order m at z_0.

ii) there is a function g, which is analytic and nonzero at z_0 such that

$f(z) = (z - z_0)^m g(z)$.

OR

c) Suppose that a function f is bounded and analytic in some deleted neighborhood $0 < |z - z_0| < \in$ of z_0. If f is not analytic at z_0, then prove that it has a removable singularity there. **10**

d) Use residues to evaluate the integral $\int_0^{\infty} \frac{\cos 2x}{\left(x^2 + 4\right)^2} dx = \frac{5\pi}{32e^4}$ **10**

UNIT – IV

4. a) Show that the mapping w = 1/z transforms circles and lines into circles and lines. **10**

b) Find a linear fractional transformation that maps the points $z_1 = 1, \ z_2 = 0$ and $z_3 = -1$ onto the points $w_1 = i, \ w_2 = \infty$ and $w_3 = 1$. **10**

OR

c) Show that the transformation w = sin z is a one to one mapping of the semi-infinite strip $-\pi/2 \le x \le \pi/2, \ y \ge 0$ in the z plane onto the upper half $v \ge 0$ of the w plane. **10**

d) Show that the image of the vertical strip $0 \le x \le 1, \ y \ge 0$ under the mapping $w = z^2$ is a closed semiparabolic region. **10**

5. a) Show that $\log\left(i^2\right) = 2 \log i$. **5**

b) Prove that the absolute convergence of a series of complex numbers implies the convergence of that series. **5**

c) Define **5**

i) Removable singular point ii) Essential singular point.

d) Show that the transformation w = iz + i maps the half plane x > 0 onto the half plane v > 1. **5**

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