M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-II

PSCMTH10: Core Elective Course : Differential Geometry

P. Pages : 3 **GUG/W/22/13750**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Prove that **10**

i) If ω is the angle between the parametric curves at the point of intersection, then $\tan\omega = \dfrac{H}{F}$.

ii) If ds represents the element of area PQRS on the surface then prove that ds = Hdudv

b) If $\left(\ell^1, m^1\right)$ are the direction coefficients of a line which makes an angle $\dfrac{\pi}{2}$ with the line whose direction coefficients are (ℓ, m) then prove that- **10**

$$\ell^1 = -\frac{1}{H}(F\ell + Gm),\ m^1 = \frac{1}{H}(E\ell + Fm)$$

OR

c) Show that the curves bisecting the angle between the parametric curves are given by $Edu^2 - Gdv^2 = 0$. **10**

d) Show that the direction of curves bisecting the angle between the orthogonal parametric curves are **10**

$$\left(\pm\frac{1}{\sqrt{2}}\frac{1}{\sqrt{E}}, \pm\frac{1}{\sqrt{2}}\frac{1}{\sqrt{G}}\right)$$

UNIT – II

2. a) If the arc length S is the parameter of the curve, then prove that the geodesic equations are **10**

$$U = \frac{d}{ds}\left(\frac{\partial T}{\partial u'}\right) - \frac{\partial T}{\partial u} = 0 \ \&$$

$$V = \frac{d}{ds}\left(\frac{\partial T}{\partial v'}\right) - \frac{\partial T}{\partial v} = 0$$

b) Show that with S as parameter, the components of the geodesic curvature vector are given by **10**

$$\lambda = \frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial V'} = -\frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial v'} \text{ and}$$

$$\mu = \frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial u'} = -\frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial u'}$$

OR

c) If U and V are the intrinsic quantities of a surface at a point (u, v) then prove that **10**

i) $K_g = \frac{1}{H}\frac{V(s)}{u'}$ and

ii) $K_g = -\frac{1}{H}\frac{U(s)}{v'}$

d) When a geodesic makes an angle θ with the curve v=constant of a geodesic coordinate system for which $ds^2 = du^2 + Gdv^2$, then prove that $\frac{d\theta}{ds} + \frac{\partial}{\partial u}\left(\sqrt{G}\right)v' = 0$ **10**

UNIT – III

3. a) Show that the anchor ring contains all three types of points namely elliptic, parabolic & hyperbolic on the surface. **10**

b) Show that the principal directions are given by **10**

$(Em - FL)\ell^2 + (EN - GL)\ell m + (FN - GM)m^2 = 0$.

OR

c) If K is the normal curvature in a direction making an angle ψ with the principal direction v = constant then prove that **10**

$K = k_a \cos^2\psi + k_b \sin^2\psi$.

d) Prove that for a ruled surface **10**

$R(u,v) = r(u) + vg(u)$, where $r = r(u)$ is

a point on the base curve and g (u) is a unit vector along the generator, the Gaussian curvature

$$K = -\frac{[\dot{r}, g, \dot{g}]^2}{H^4}$$

UNIT – IV

4. a) If $N_1 = \frac{\partial N}{\partial u}$ and $N_2 = \frac{\partial N}{\partial v}$ then prove that **10**

i) $N_1 = \frac{1}{H^2}\left[(FM - GL)r_1 + (FL - EM)r_2\right]$ and

ii) $N_2 = \frac{1}{H^2}\left[(FN - GM)r_1 + (FM - EN)r_2\right]$

b) If the lines of curvature are parametric curves then obtain the Codazzi equations. **10**

i) $L_2 = \frac{1}{2}E_2\left(\frac{L}{E} + \frac{N}{G}\right)$ & ii) $N_1 = \frac{1}{2}G_1\left(\frac{L}{E} + \frac{N}{G}\right)$

OR

c) If K_a & K_b are the principal curvatures, then prove that Codazzi equations are- **10**

$$(K_a)_2 = \frac{1}{2}\frac{E_2}{E}(K_b - K_a) \text{ and}$$

$$(K_b)_1 = \frac{1}{2}\frac{G_1}{G}(K_a - K_b)$$

d) Prove that the parallel surfaces of a minimal surface are surfaces for which $R_a + R_b =$ constant where $R_a = \frac{1}{K_a}$ & $R_b = \frac{1}{K_b}$. **10**

5. a) Find E, F, G & H for the paraboloid. **5**
$x = u,\ y = v,\ z = u^2 - v^2$.

b) Find the Gaussian curvature at every point of a sphere of a radius a **5**

c) Find L, M, N for the sphere **5**
$r = (a\cos u \cos v,\ a \cos u \sin v,\ a \sin u)$ where u is the latitude & v is the longitude.

d) Show that only closed surface of constant positive curvature without singularities is a sphere. **5**

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