M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-II

PSCMTH09 / MSCMTH09 : Classical Mechanics

P. Pages : 2 **GUG/W/22/13749**

Time : Three Hours * 3 4 5 3 * Max. Marks : 100

Notes : 1. All questions carry equal marks.
2. Solve all the **five** questions.

UNIT – I

1. a) State & obtain the solution for Brachistochrone problem. **10**

b) Obtain the equation of catenary by the minimum surface of revolution. **10**

OR

c) Derive the Lagrange's equations from the Hamilton's principle. **10**

d) Discuss the extension of Hamilton's principle to nonholonomic system. **10**

UNIT – II

2. a) Show that the momentum conjugate to the time coordinate is the negative of the ordinary Hamiltonian. **10**

b) Discuss the Routh's procedure & show that this nonignorable coordinates obey the Lagrange equation **10**

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_i}\right)-\frac{\partial R}{\partial q_i}=0,\ i=1,\ 2,\ \ldots\ldots,s$$

With R as a Lagrangian.

OR

c) Derive Hamilton's equations from a variational principle. **10**

d) Obtain the Hamilton's canonical equations. **10**

UNIT – III

3. a) Show that symplectic condition holds for any infinitesimal transformations. **10**

b) Obtain the equations of canonical transformations. **10**

OR

c) Show that fundamental Poisson brackets are invariant under canonical transformation. **10**

d) Obtain the equation **10**

$$p_i\,\dot{q}_i - H = P_i\,\dot{Q}_i - k + \frac{df}{dt}$$

UNIT – IV

4. a) Discuss the symmetric group of mechanical system. **10**

b) Show that the constant of motion are generating functions of those infinitesimal transformation that leave the Hamiltonian invariant. **10**

OR

c) Obtain the angular momentum Poisson bracket relations. **10**

d) Show that the density of the system in the neighborhood of some given system in phase space remains constant in time **10**

i.e. $\frac{dD}{dt} = 0$ or $\frac{\partial D}{\partial t} = -[D, H]$.

5. a) Show that the shortest distance between two points in a plane is a straight line. **5**

b) Show that : $\frac{dH}{dt} = \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$. **5**

c) State the example of canonical transformation which merely generates the identity transformation. **5**

d) Obtain the relations: **5**

$\dot{q}_i = [q_i, H]$, $\dot{p}_i = [p_i, H]$.

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