M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-II

PSCMTH08: : Advanced Topics in Topology

P. Pages : 2

Time : Three Hours

GUG/W/22/13748

Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that a topological space X is completely normal iff every subspace of X is normal. **10**

b) Prove that every separable metric space is second axiom. **10**

OR

c) Prove that a normal space is completely regular iff it is regular. **10**

d) Prove that every countably compact metric space is totally bounded. **10**

UNIT – II

2. a) Prove that the projections Π_x & Π_y are continuous & open mappings but not necessarily closed, and the product topology is the smallest topology for which the projections are continuous. **10**

b) Prove that $\prod_{\lambda} X_\lambda$ is connected iff each space X_λ is connected. **10**

OR

c) Prove that XxY is dense-in-itself iff at least one of the spaces X & Y is dense-in-itself. **10**

d) Prove that $\Pi_\lambda X_\lambda$ is Hausdorff iff each space X_λ is Hausdorff. **10**

UNIT – III

3. a) Prove that a subset G of Y is open in the quotient topology relative to $f : x \to y$ iff $f^{-1}(G)$ is an open subset of X. **10**

b) Prove that every paracompact regular space is normal. **10**

OR

c) If X is locally connected, prove that Y is locally connected with the quotient topology. **10**

d) Prove that every second axiom T_3-space is metrizable. **10**

UNIT – IV

4. a) Prove that A topological space is Hausdorff iff limits of all nets in it are unique. **10**

b) Prove that A topological space is Hausdorff iff no filter can converge to more than one point in it. **10**

OR

c) Let A be a subset of a space X & let $x \in X$. Then prove that $x \in \overline{A}$ iff there exists a net in A which converges to x in X. **10**

d) For a topological space X, prove that the following statements are equivalent: **10**
- i) X is compact
- ii) Every net in X has a cluster point in X
- iii) Every net in X has a convergent subnet in X

5. a) Prove that every metric space is a Hausdorff space. **5**

b) Define- **5**
- i) Tichonov Topology
- ii) Projections

c) If f is a continuous, open mapping of the topological space X onto the topological space Y, then prove that the topology for Y must be the quotient topology. **5**

d) Define- **5**
- i) Directed Set
- ii) Filter

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