M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-I

PSCMTH05 (D) - Optional Paper : Number Theory

P. Pages : 2 **GUG/W/22/13744**

Time : Three Hours * 3 4 4 8 * Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Let f denote a polynomial with integral coefficients. If $a \equiv b \pmod{m}$ then prove that $f(a) \equiv f(b) \pmod{m}$. **10**

b) Let p be a prime number. Then prove that $x^2 \equiv 1 \pmod{p}$ iff $x \equiv \pm 1 \pmod{p}$. **10**

OR

c) If p is prime, then prove that $(p-1)i \equiv -1 \pmod{p}$ **10**

d) Solve the congruence **10**
$x \equiv 5 \pmod{7},\ x \equiv 7 \pmod{11},\ x \equiv 3 \pmod{13}$

UNIT – II

2. a) Solve $x^2 + x + 7 \pmod{81}$ **10**

b) Solve $x^2 + x + 47 \equiv 0 \pmod{7^3}$ **10**

OR

c) If a has order h modulo m, then prove that a^k has order $h/(h,k)$ modulo m. **10**

d) If p is a prime then prove that there exist $\oint(p-1)$ primitive roots modulo p. **10**

UNIT – III

3. a) Let p denote an odd prime & a an integer relatively prime to p then prove that **10**

i) $\left(\frac{a}{p}\right) = a^{(p-1)/2} \pmod{p}$

ii) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$

b) Define: **10**
i) Quadratic residues modulo m
ii) Quadratic non residues modulo m
Prove that 3 is a quadratic residue of 13 & quadratic non-residue of 7.

OR

c) If p is an odd prime and $(a, 2p) = 1$, then prove that $\left(\frac{a}{p}\right) = (-1)^t$ **10**

where $t = \sum_{j=1}^{\left(\frac{p-1}{2}\right)} \left[\frac{ja}{p}\right]$; also $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$.

d) Let x & y be real numbers, then prove that **10**

i) $[x]+[y] \le [x+y] \le [x]+[y]+1$

ii) $[x]+[-x] = \begin{cases} 0, & \text{if } x \text{ is an integer} \\ -1, & \text{otherwise} \end{cases}$

iii) $\left[\frac{[x]}{m}\right] = \left[\frac{x}{m}\right]$ if m is a positive integer

UNIT – IV

4. a) Define Mobius mu function & prove that if $F(n) = \sum_{d/n} f(d)$ for every positive integer n, then $f(n) = \sum_{d/n} \mu(d) f(n/d)$. **10**

b) Find all solutions of $999x - 49y = 5000$. **10**

OR

c) Find all solutions in positive integers $15x + 7y = 111$. **10**

d) Find all Pythagorean triples whose terms form an arithmetic progression. **10**

5. a) If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ then prove that $a \equiv c \pmod{m}$. **5**

b) Prove that if $(a, m) = 1$, then the order of a modulo m divides $\phi(m)$. **5**

c) Let p is a prime integer, let $(a, p) = 1$ & $(b, p) = 1$ if $x^2 \equiv a \pmod{p}$ & $x^2 \equiv b \pmod{p}$ are not solvable, then prove that $x^2 \equiv ab \pmod{p}$ is solvable. **5**

d) Find all primitive solutions of $x^2 + y^2 = z^2$ having $0 < z < 30$. **5**

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