M.Sc.(Mathematics) (New CBCS Pattern) Semester - I

# Elective Course Code : PSCMTH05(A) - Numerical Analysis Paper-V

P. Pages : 2 **GUG/S/23/13741**

Time : Three Hours Max. Marks : 100

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Notes : 1. Solve **all five** questions.
2. Each questions carries equal marks.

## UNIT - I

**1.** a) Assume $f(x), f'(x)$ and $f''(x)$ are continuous for all x in some neighbourhood of $\alpha$, and assume $f(\alpha)=0, f'(\alpha)\neq 0$ then prove that if $x_0$ is chosen sufficiently close to $\alpha$ the iterates $x_n, n\geq 0$ of $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$, will converge to $\alpha$. **10**

b) Discuss the secant method and prove convergence of $x_n$ to $\alpha$ under suitable condition. **10**

**OR**

c) Consider Newton's method for finding the positive square root of $a>0$. Derive the following result, assuming $x_0>0, x_0\neq\sqrt{a}$. **10**

i) $x_{n+1}=\dfrac{1}{2}(x_n+a/x_n)$ ii) $x_{n+1}^2-a=\left[\dfrac{x_n^2-a}{2x_n}\right]^2, n\geq 0$

iii) The iterates $\{x_n\}$ are strictly decreasing sequence for $n\geq 1$.

iv) $e_{n+1}=-e_n^2/2x_n$ with $e_n=\sqrt{a}-x_n$.

d) Apply Newton's method to the following function. **10**

$$f(x)=\begin{cases}\sqrt{x}, & x\geq 0\\ -\sqrt{-x}, & x\leq 0\end{cases}$$

with root $\alpha=0$, what is the behaviour of the iterates? Do they converage and if so, at what rate?

## UNIT - II

**2.** a) Show that for any two functions f and g and for any two constants $\alpha$ and $\beta$. **10**

$$\Delta^r(\alpha f(x)+\beta g(x))=\alpha\Delta^r f(x)+\beta\Delta^r g(x), r\geq 0$$

b) Find the Hermite interpolating polynomial for which **10**

$$p(a)=f(b),\ p'(a)=f'(a)$$
$$p(b)=f(b),\ p'(b)=f'(b)$$

**OR**

c) Let $x_0,....,x_n$ be distinct real numbers and let f(x) be n times continuously differentiable on interval $H\{x_0,.....,x_n\}$ then show that- **10**

$$f[x_0,.....,x_n]=\int\int.....\int_{T_n} f^{(n)}(t_0x_0+....+t_nx_n)\,dt_1.......dt_n$$

$$T_n=\left\{(t_1,.......,t_n)\Big/\ t_1\geq 0,........t_n\geq 0, \sum_i^n t_i\leq 1\right\},\ t_0=1-\sum_1^n t_i$$

d) Prove that for $k \geq 0$ **10**

$$f[x_{0,}......, x_k] = \frac{1}{k!\, h^k} \Delta^k f_0$$

where $f_0 = f(x_0)$ & $f_i = f(x_i)$

## UNIT - III

**3.** a) To obtain a minimax polynomial approximation $a_1^*(x)$ for the functions $f(x) = e^x$ on the interval [-1, 1]. **10**

b) If $\{\phi_n(x) | n \geq 0\}$ is an orthogonal family of polynomials on (a, b) with weight function $\omega(x) \geq 0$. Then prove that the polynomial $\phi_n(x)$ has exactly n distinct real roots in the open interval (a, b) **10**

**OR**

c) Discuss the Gram-Schmidt theorem. **10**

d) Find linear least square approximation of the function $f(x) = e^x$ on $-1 \leq x \leq 1$. **10**

## UNIT - IV

**4.** a) Obtain simple trapezoidal rule with error. **10**

b) Obtain simple Simpson's rule of integration, obtain error estimate. **10**

**OR**

c) For n even, assume $f(x)$ is $n+2$ times continuously differentiable on [a, b] then prove that $I(f) - I_n(f) = c_n h^{n+3} f^{(n+2)}(n)$ some $n \in [a, b]$ **10**

with $c_n = \dfrac{1}{(n+2)!} \displaystyle\int_0^n \mu^2 (\mu - 1).....(\mu - n)\, d\mu$.

d) Derive Newton-Cotes integration formula for $n = 1$. **10**

**5.** a) Apply Newton's method to the function **5**

$$f(x) = \begin{cases} \sqrt[3]{x^2}, & x \geq 0 \\ -\sqrt[3]{x^2}, & x \leq 0 \end{cases}$$

with root $\alpha = 0$, what is the behaviour of iterates? Do they converge, and if so, at what rate?

b) Obtain the expression for $p_1(x)$ by Lagrange interpolation. **5**

c) For $f, g \in c[a, b]$ then prove that $\|f + g\|_2 \leq \|f\|_2 + \|g\|_2$ **5**

d) Discuss the open Newton-Cotes formula. **5**

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