M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-I

PSCMTH05 (A) - Optional Paper : Numerical Analysis

P. Pages : 2 **GUG/W/22/13741**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** question.
2. Each question carries equal marks.

UNIT – I

1. a) Let f(x) $f'(x), f''(x)$ are continuous for all value of x in some interval containing α, and assume $f'(\alpha) \neq 0$ then prove that if the initial guesses x_0 and x_1 are chosen sufficiently close to α, the iterates x_n of $x_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$, $n \geq 1$ will converge to α the order of convergence will be $P = (1+\sqrt{5})/2 \simeq 1.62$ **10**

b) Assume f(x), $f'(x)$ $f''(x)$ are continuous for all x in some neighbourhood of α and assume $f(\alpha) = 0$, $f'(\alpha) \neq 0$ then prove that if x_0 is chosen sufficiently close to α the iterates $x_n, n \geq 0$ of $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, will converge to α **10**

OR

c) Apply the Newton's method for the function **10**

$$f(x) = \sqrt{x} \quad , x \geq 0$$
$$= -\sqrt{-x} \quad , x < 0$$

With root $\alpha = 0$, what is the behavior of the interates? Do they converge, and if 50 at what rate?

d) Discuss Muller's method for finding roots of a polynomial. Discuss why Muller's method is better than the secant method. **10**

UNIT – II

2. a) Define n^{th} order of Newton's divided difference of a function of namely **10**

$$f[x_0, ----, x_n] = \frac{f(x_1, ----x_n) - f[x_0, ----, x_{n-1}]}{x_n - x_0}$$

b) Prove that for $k \geq 0$ **10**

$$f[x_0, x_1, ----x_k] = \frac{1}{k!h^k} \Delta^k f_0 \quad \text{where } f_0 = f(x_0) \ \& \ f_i = f(x_i)$$

OR

c) For any two functions f and g and for any two constant's α and β **10**

$$\Delta^r(\alpha f(x) + \beta . g(x)) = \alpha \Delta^r f(x) + \beta \Delta^r g(x), r \geq 0$$

d) For the basis functions $\ell_{j,n}(x)$ given by $\ell_i(x) = \prod_{j \neq i} \left(\frac{x - x_j}{x_i - x_j} \right) i = 1, 2 ----h$ **10**

Then prove that for $n \geq 1$, $\sum_{j=0}^{h} \ell_{j,n}(x) = 1$ for all x

UNIT – III

3. a) Let f(x) be continuous for $a \leq x \leq b$ and let $\in 70$, then prove that there is a polynomial p(x) for which **10**

$|f(x) - p(x)| \leq t, \qquad a \leq x \leq b$

b) Find the liner least square approximation of the function f(x) = e^x, $-1 \leq x \leq 1$ **10**

OR

c) Prove that, for **10**

$f, g \in \subset [a, b]$,

$|(f, g)| \leq \| f \|_2 \| g \|_2$

d) To obtain a minimax polynomial approximation $a_1^*(x)$ for the function $f(x) = e^x$ on the interval [-1,1] **10**

UNIT – IV

4. a) Obtain the composite trapezoidal rule with error. Find the expression for the asymptotic error. **10**

b) Obtain the expression for Peano-Kernel error formula. **10**

OR

c) Obtain the formula for the simple Simpson's rule of integration, obtain error estimate. **10**

d) Derive Newton-cotes integration formula for n = 1 **10**

5. a) Show that, **5**

If g(x) be continuous for $a \leq x \leq b$ and assume that $a \leq g(x) \leq b$ for $a \leq x \leq b$ then x = g(x) has at least one solution in [a, b]

b) Obtain the expression for $P_1(x)$ by Lagrange interpolation **5**

c) For $f, g \in C[a, b]$ then prove that $\| f + g \|_2 \leq \| f \|_2 + \| g \|_2$ **5**

d) Obtain simple trapezoidal rule **5**

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