M.Sc.(Mathematics) (New CBCS Pattern) Semester - I

PSCMTH03 - Topology-I

P. Pages : 2

GUG/S/23/13739

Time : Three Hours

Max. Marks : 100

Notes : 1. All five questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the set of all real numbers is uncountable. **10**

b) Prove that every infinite set contains a denumerable subset. **10**

OR

c) Prove that **10**

i) $N_0 N_0 = N_0$

ii) $N_0 c = c$

iii) $cc = c$

Where N denotes Hebrew letter aleph.

d) Prove that the union of a denumerable number of denumerable sets is a denumerable set. **10**

UNIT – II

2. a) If A, B are subsets of the topological space (X, τ), then prove that the derived set has the following properties: **10**

i) If $A \subseteq B$, then $d(A) \subseteq d(B)$

ii) $d(A \cup B) = d(A) \cup d(B)$

b) Prove that a set F is a closed subset of a topological space iff F^c is an open subset of the space. **10**

OR

c) Prove that for any set E in a topological space **10**

$\subset(E) = E \cup d(E)$

d) Let $X = \{a, b, c\}$ & let **10**

$\tau = \{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}$

Find

i) $d(\{b\})$

ii) $d(X)$

UNIT – III

3. a) If C is a connected set and $C \subseteq E \subseteq \subset(C)$, then prove that E is a connected set. **10**

b) If E is a subset of a subspace $\left(X^{*}, \tau^{*}\right)$ of a topological space (X, τ), then prove that E is τ^{*} - compact iff it is τ - compact. **10**

OR

c) If F is a continuous mapping of (X, τ) into $\left(X^{*}, \tau^{*}\right)$, then prove that F maps every connected subset of X onto a connected subset of X^{*} **10**

d) Prove that a mapping F of X into X^{*} is open iff $F(i(E)) \subseteq i^{*}(F(E))$ for every $E \subseteq X$. **10**

UNIT – IV

4. a) Prove that a topological space X is a T_1 - space iff every subset consisting of exactly one point is closed. **10**

b) Prove that in a Hausdorff space, a convergent sequence has a unique limit. **10**

OR

c) Prove that in a second axiom space, ever collection of nonempty disjoint, open sets is countable. **10**

d) Prove that a topological space X is normal iff for any closed set F and open set G containing F, there exists an open set G^{*} such that $F \subseteq G^{*}$ and $C\left(G^{*}\right) \subseteq G$ **10**

5. a) Prove that every infinite set is equipotent to a proper subset of itself. **5**

b) Define **5**
   i) Topological space.
   ii) Limit point.

c) Define: **5**
   i) Open mapping.
   ii) Closed mapping.
   iii) Homeomorphism.

d) Define: **5**
   i) First axiom space.
   ii) Second axiom space.

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